



Signal Integrity Issues and High-Speed Interconnects

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Transactions on CPMT

- 595 submissions (2020)
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Technical Program (data as of last year)

		Ave	rage	Electron	nics Packaging	
# of PDFs Publish	ed		8,469		6,822	
# of Events			76		74	
# of Articles View	ved (Downloads)	1	1,666,555 1,395,158		1,395,158	
PDFs/Conference	2	112		92		
Downloads/PDF			197		204	
Avg OU Package	Net		1.5%		1.3%	
Financial Spon	sor Co Sponsor	Techr	nical	Spor	nsor	
► ECTC	► EPEPS	ASMC		► IIV	IPACT	
EDAPS	► EPTC	DTIP		► ISS	SE	
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EPS Awards & Recognition

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Outstanding Sustained Technical Contribution Award

Electronics Manufacturing Technology

David Feldman Outstanding Contribution

Exceptional Technical Achievement

Outstanding Young Engineer

l Contributions

Transactions Best Papers

Regional Contributions

PhD Fellowship





Carleton University – Canada's Capital University



Research @ CAD Group in Carleton

- High-Speed Interconnects,
- Signal & Power Integrity
- Circuit Simulation
- Timing Analysis
- Model-Order Reduction Algorithms
- Variational Analysis
- Optimization
- Mixed Digital, Analog, EM, RF Analysis
- Parallel Algorithms
- Neural Networks

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Design Trends

- Faster Devices
- Compact Products
- Multi-Function
- Less Clutter
- Use Less Power

- High-Frequency
- High-Density
- Wireless
- Low-Power



Agenda

- Emerging Product Trends
- Interconnect Scaling
- Signal Integrity Issues
- Interconnect Hierarchy
- What is a "High-Speed Interconnect"?
- Interconnect Models and Simulation Challenges
- Advanced/Recent Models
- Conclusions

High-Speed Design Issues



Impact of Signal Integrity Issues



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Moore's Law on Density/Speed



intel

Source: Intel, www.intel.com

Evolution of Density and Frequency

- In a Billion Transistor Design, there could be multiple billions of interconnects, many of which do not scale in performance
- High-Speed Signal Propagation Issues

Transistor, Interconnect – Scaling & SI Issues



Technology Scaling:

Scale W, L & t_{ox} by a factor of 'S'

Transistor, Interconnect – Scaling & SI Issues

Parameter	Relation	Scaling Factor
Dimensions	W, L, t _{ox}	1/S
Voltages	V_{DD}, V_{T}	1/S
Currents	I _{DS}	1/S
Power Dissipation/Gate	P = IV	1/S ²
Area Per Device	A = WxL	1/S ²
Power Dissipation Density	P/A	1

Transistor - Ideal Scaling

Impact on Transistor Related Delays

Parameter	Relation	Scaling Factor
Gate Capacitance	$C_g = (\epsilon_{ox}/t_{ox})(W \times L)$	1/S
Transistor On Resistance	$R_{tr} = V_{DD} / I_{DS}$	1
Intrinsic Gate Delay	$T_g = R_{tr} C_g$	1/S

A look at Interconnects.....



Scaling: Local Interconnections

Parameter	Relation	Scaling Factor
Cross-sectional Dimensions	W _{int} , H _{int} , W _{sp} , T _{int}	1/S
Capacitance Per Unit Length	C _{int} α (ε _{ox})(W _{int} /t _{ox})	1
Resistance Per Unit Length	$R_{int} = \zeta_{int} / (W_{int} \times H_{int})$	S ²
RC Time Constant per unit Length	R _{int} x C _{int}	S ²
Local Interconnection Length	L _{int}	1/S
Total Local Interconnection RC Delay	$T = (R_{int} C_{int}) L^{2}_{int}$	1

Scaling: Global Interconnections

Parameter	Relation	Scaling Factor
Die Size		S _c
Global Interconnection Length	L _{glob}	S _c
Global Interconnections RC Delay	$T = (R_{int} C_{int}) L^2_{glob}$	S ² S ² _c

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Attenuation



Interconnect Effects: Delay



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Reflection/Ringing



Crosstalk





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Ubiquitous Interconnects



Role of the Package

- Distribute power and signals,
- Dissipate the heat generated by the IC.
- Mechanical support for the chip,

Issues to tackle, parasitic elements such as:

- capacitive coupling between connections or leads,
- inductance of the connections or leads,
- resistance of the connections
- The values of the parasitic elements depend on the package layout and structure,
 Have Significant Impact on the Package Performance.

Role of the Package

- The design and construction of packages vary significantly, but most of them are fabricated either from plastic or ceramic materials.
- The chip to package interconnects can be divided into three main categories:

Wire-bond (WB) Tape-automated-bond (TAB) Flip-chip

Package – Wire Bond

Wire-bond (WB): Although the oldest method, wire-bonding is still the dominant packaging method used today.





Due to the self-inductance and mutual inductance of the wires, noise on the PDN and crosstalk between adjacent signals might occur.

Ground Bounce

On-board VCC bus



On-board ground bus

$V_{g} = L_{g} \times \frac{d [i_{discharge}(t) \ total]}{dt}$

Flip Chip Technology



-Uniform Distribution of Power -Shorter contacts: reduced Parasitics -More Widely used
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High-Speed!! What is it??

- Time taken to travel through the interconnect can no longer be neglected!!
- Interconnects can no more be treated as electrically short ($\rightarrow d \le \lambda/10$)
- Need to start worrying about high-frequency effects when:

$d \ge \lambda/10 \rightarrow$ Electrically Long Interconnects

What is a High-Speed Interconnect?





At higher frequencies, Interconnect length becomes comparable to the Wavelength



Frequency = 1GHz
$$\rightarrow \lambda \approx \frac{\mathbf{v}}{\mathbf{f}} = \frac{1.5 \times 10^{10}}{1 \times 10^9} = 15 \text{ cm}$$

 $\rightarrow d > 1.5 \text{ cm}$

Time-Freq Relations



What is an High-Speed Interconnect?

Sharper pulses contain higher frequency harmonics

$$f_{\text{max}} = \frac{0.35}{t_r \sqrt{t_r}}$$

$t_r = 0.1ns \rightarrow f_{max} = 3.5GHz \rightarrow$

→ d < 4mm

$$\lambda \approx \frac{\mathbf{v}}{\mathbf{f}} = \frac{1.5 \times 10^{10}}{3.5 \times 10^9} \approx 4 \text{ cm}$$

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Modeling of Interconnects



Interconnect Models



Distributed Transmission Lines

Distributed Transmission Lines

- ➔ Lossless, Lossy
- → Single, Multiconductor
- Frequency Independent/ dependent
 - p.u.l. parameters
 - (skin/proximity/edge effects)
- → Uniform/Non-uniform
- Current Distribution Related Effects:
 - Skin Effect
 Edge Effect
 Proximity Effect
- Surface Roughness Effects

Frequency Dependence of R & L Parameters



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Multi-Conductor Transmission Lines



Transient Analysis

Mixed Frequency/Time Simulation



Lumped Segmentation - Large Circuit



Large CPU Cost

Transient Simulation Issues

Mixed Frequency/Time

- Complexity
- CPU time
- Memory

Simulator Interface

MACROMODELING

 $\frac{\partial}{\partial z} V(z,t) = -R I(z,t) - L \frac{\partial}{\partial t} I(z,t)$ $\frac{\partial}{\partial z} I(z,t) = -G V(z,t) - C \frac{\partial}{\partial t} V(z,t)$

Macromodeling

Circuit
Simulators \int $\frac{d}{dt}x = Ax + Bu$
y = Cx

Macromodeling

- Lumped Segmentation
- Method of Characteristics
- Least-Square Optimization
- Chebeshev, Wavelet Polynomials
- Compact Finite Differences
- Integrated Congruent Transformation
- Matrix Rational Approximation
- Model-Order Reduction Methods....

Discretization



Lumped Models - Pi

Lumped LC Lossless Line



Lumped RLGC Lossy Line



Lumped Cascaded Lossless Distributed Line









50 segments v/s Distributed Model



Transient Analysis with Distributed Interconnect



Frequency Response of Lumped Models



Frequency Response: Lumped Model with Input Pulse Spectrum



Transient Response



A practically used expression to determine the number of sections (N) needed is given by:

$$N = (10\tau d)/t_r \qquad t_r = risetime$$

$$\tau = line \ delay$$

$$d = line \ length$$

Example: Rise time = 0.2ns; Lossless line: 10cm, per-unit delay =of 70.7ps.

$$N = (10 \times 70.7e^{-12} \times 10) / (0.2e^{-9}) = 35$$

Direct Lumped Segmentation



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 - > MRA, DEPACT
 - > WR+TP, WR+TP+EMI
 - Tabulated Data, Parallelization
- Conclusions

Possible Efficient Macromodeling Approaches

1) MoC based Algorithms

2) Matrix Rational Approximations

MoC based Algorithms

→ Delay Extraction + Rational Approximation

Efficient for Long Low Loss Lines

Difficulties

1) Coupled Lines: Curve Fitting
 n Lines → (2n² + n) Tr. functions
 Eg. 10 Lines → 210 Tr. functions

2) Does not Guarantee Passivity



- Limited Bandwidth of Approximation
- Individual numerical fitting of parameters of a matrix function
- Loss of physical properties such as passivity

Importance of Passivity



Error: Failure to Converge Error: Time Step Too Small – Abort Error:

Unstable response



Time response of stable but nonpassive reduced model

Passivity



Y(s) is passive iff **1)** $Y(s^*) = Y^*(s)$ 2) $z^{*t}[Y^t(s^*) + Y(s)]z \ge 0$, Re(s)>0 Y(s) is a positive real matrix **Ensuring passivity of the reduced Macromodel is a challenging task!!**

Defining the objectives.....

 Can we improve the bandwidth of approximation without facing ill-conditioning?

 Can we come up with a matrix based approximation without resorting to individual entity approximations?

 Can we do the approximation analytically without resorting to numerical curve fitting?

Can we ensure the physical properties for the model?
<u>Agenda</u>

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$$\frac{\partial}{\partial z} \mathbf{V}(z,t) = -\mathbf{R} \mathbf{I}(z,t) - \mathbf{L} \frac{\partial}{\partial t} \mathbf{I}(z,t)$$
$$\frac{\partial}{\partial z} \mathbf{I}(z,t) = -\mathbf{G} \mathbf{V}(z,t) - \mathbf{C} \frac{\partial}{\partial t} \mathbf{V}(z,t)$$
$$\begin{bmatrix} \mathbf{I}(d,s) \\ \mathbf{V}(d,s) \end{bmatrix} = e^{\mathbf{Z}} \begin{bmatrix} \mathbf{I}(0,s) \\ \mathbf{V}(0,s) \end{bmatrix}$$
$$\mathbf{Z} = (\mathbf{D} + s\mathbf{E})d \qquad \mathbf{D} = \begin{bmatrix} \mathbf{0} & -\mathbf{R} \\ -\mathbf{G} & \mathbf{0} \end{bmatrix} \qquad \mathbf{E} = \begin{bmatrix} \mathbf{0} & -\mathbf{L} \\ -\mathbf{C} & \mathbf{0} \end{bmatrix}$$

This is what we are approximating.....

Matrix Rational Approximation



Closed-Form Relations
 Can achieve higher bandwidth

CONCEPT - MRA

Pade' Approximation of Exponential Function



→Closed-form relation for coefficients
→High-order approximation possible
→No ill-conditioning

Time-Domain Macromodel



Passivity??



Time response of stable but nonpassive reduced model

Passivity Conditions

<u>Theorem</u>: Let the rational function of *e*^x be:

$$\mathbf{e}^{\mathbf{x}} \approx \frac{Q_{N}(\mathbf{x})}{Q_{N}(-\mathbf{x})} = \frac{\sum_{i=0}^{N} q_{i} \mathbf{x}^{i}}{\sum_{i=0}^{N} q_{i} (-\mathbf{x})^{i}}$$

IF the polynomial $Q_N(x)$ is strictly Hurwitz^{*} THEN

the rational matrix obtained by replacing the scalar x with the matrix Z=(D+sE)d results in a passive transmission line macromodel

*A <u>polynomial</u> with <u>real positive</u> <u>coefficients</u> and <u>roots</u> which are either <u>negative</u> or pairwise conjugate with <u>negative</u> <u>real parts</u>.

Example 1: Coupled Lossy TL



Input: step response, rise time = 0.2 ns

Example 1: Far End Active Line



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Performance Comparison

Simulations	MRA	Lumped	MNA
	(MNA size)	(MNA size)	savings
Example 1	8281	48000	83%
Example 2	355	2 482	86%
Example3 (5cm)	914	6 002	85%
Example 3 (20cm)	3 650	24 002	85%
Example 3 (40cm)	7 298	80 002	91%

Example 3: Nonlinear Terminations



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Transient Responses



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CPU Comparison

Algorithm	Total number of lumped sections	CPU time (SPARC Ultra 5-10) (seconds)
Conventional Lumped	300	3282
Proposed	31	315

How About Long Low-Loss Lines???



Why?

Without Delay Extraction



Macromodel: Objectives

1) Closed Form

Large Number of Coupled Lines

2) Guaranteed Passivity

3) Delay Extraction



Advanced Interconnect Modeling Methods

- MoC, MRA
 DEPACT

 - WR+TP, EMI
 - Tabulated Data Macromodelling, Parallelization

Conclusions

Delay Extraction - DEPACT

 Extracts pure delay components of equivalent circuit matrix

 Pure-delay components can be simulated with existing tline (T)

Highly compact/passive macromodels

Error Estimates



Error Estimates



Example: Lossy Coupled TL

5cm, 20cm, 40cm



Input: step response, rise time = 0.035 ns

IBM: Line 6 (40cm)



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IBM: Line 6

Simulation	DEPACT	MRA	Lumped
	time (s)	time (s)	time (s)
5cm	0.89	4.16	32.4
20cm	2.47	25	292
40cm	4.32	74	4641

Computer: SUN Blade-1000 workstation with 900MHz UltraSPARC-III CPU.

Problem Addressed so far.....

- Limited Bandwidth of Approximation
- Individual numerical fitting of parameters of a matrix function
- Loss of physical properties such as passivity
- Delay Extraction + Passivity

How about large coupled lines?



Advanced Interconnect Modeling Methods

MoC, MRA, DEPACT
WR+TP
EMI
Tabulated Data

Conclusions

Large # of Coupled Lines: Traditional approaches



Large # of Coupled Lines



Average cost of simulating N-coupled lines circuit is proportional to N^{\alpha} , where $3 \le \alpha \le 4$

In the literature.....





WR + TP of Unexcited Lines- WR Sources



Computation of WR Source

Homogeneous Telegrapher's Equation

$$\frac{\partial}{\partial z} \begin{bmatrix} \mathbf{V}(z,t) \\ \mathbf{I}(z,t) \end{bmatrix} = \begin{bmatrix} \mathbf{0} & -\mathbf{R} \\ -\mathbf{G} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}(z,t) \\ \mathbf{I}(z,t) \end{bmatrix} + \begin{bmatrix} \mathbf{0} & -\mathbf{L} \\ -\mathbf{C} & \mathbf{0} \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} \mathbf{V}(z,t) \\ \mathbf{I}(z,t) \end{bmatrix}$$
for kth line
$$\frac{\partial}{\partial z} v_k(z,t) = -R_{kk} i_k(z,t) - L_{kk} \frac{\partial}{\partial t} i_k(z,t) - \sum_{\substack{j=1\\j\neq k}}^{n} \frac{\mathbf{e}_k(z,t)}{\mathbf{e}_k(z,t)} = \frac{\partial}{\partial t} i_k(z,t) - \frac{\partial}{\partial t} \frac{\partial}{\partial t} v_k(z,t) - \sum_{\substack{j=1\\j\neq k}}^{n} \frac{\partial}{\partial t} \frac{\partial}{\partial t} \left(z, t \right)$$

TRANSVERSE PARTITIONING + WR



Electromagnetic Interference



EMI Analysis & High-Speed Interconnects



Conventional EMI Analysis: Large # of Coupled Lines



Simulation Time→O((n²)²)=O(n⁴) !!!

WR+TP + EMI

Description of EMI:

Wa

$$\frac{\partial}{\partial z} \begin{bmatrix} \mathbf{V}(z,s) \\ \mathbf{I}(z,s) \end{bmatrix} = \begin{bmatrix} \mathbf{0} & -(\mathbf{R}+s\mathbf{L}) \\ -(\mathbf{G}+s\mathbf{C}) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}(z,s) \\ \mathbf{I}(z,s) \end{bmatrix} + \begin{bmatrix} \mathbf{V}_F(z,s) \\ \mathbf{I}_F(z,s) \end{bmatrix}$$

mutually compatible representation POSSIBLE!

$$\frac{\partial}{\partial z}v_{k}(z,t) = -R_{kk}i_{k}(z,t) - L_{kk}\frac{\partial}{\partial t}i_{k}(z,t) - \tilde{e}_{k}(z,t)$$
$$\frac{\partial}{\partial z}i_{k}(z,t) = -\hat{G}_{kk}v_{k}(z,t) - \hat{C}_{kk}\frac{\partial}{\partial t}v_{k}(z,t) - \tilde{q}_{k}(z,t)$$
Combined representation for each iteration



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Example: WR+TP + EMI

Example 2: 3 Line, Active terminations, Trapezoidal Incidence



Example: WR+TP + EMI



Example: WR+TP + EMI



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Example: Complexity Analysis



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Advanced Interconnect Modeling Methods

- > MoC, MRA, DEPACT
- > WR+TP. EMI
- Tabulated Data Macromodeling:
 - ➢ Parallelization, GVF

Conclusions

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Why measured/simulated data

At high frequencies, many complex electrical devices may have no analytical models

Example: 3D transmission lines, vias, packages non-uniform transmission lines on-chip passive devices

Characterized by tabulated data in terms of multiport Terminal parameters To identify a system for circuit simulations



Impedance parameters or Z-parameters
Admittance parameters or Y-parameters
Hybrid parameters or H-parameters
Scattering parameters or S-parameters

System identification via Direct Curve Fitting



System identification via Direct Curve Fitting

System identification via Direct Curve Fitting

ill-conditioned

Can not achieve Higher-Order Approximations

Solution -> Vector fitting algorithm

Original paper: B. Gustavsen and A. Semlyen, "Rational Approximation of Frequency Domain Responses by Vector Fitting, " *IEEE Transactions on Power Delivery*, vol. 14, no. 3, pp. 1052-1061, July 1999.

$$f(s) = \sum_{n=1}^{N} \frac{\tilde{r}_n}{s - p_n} + d + se$$

Continuously refined and evolved over last 20+ years

VF – an Iterative Algorithm

Assume an initial set of poles and a scaling function $\sum_{n=1}^{N} \tilde{r}_{n}$



Step1:Computation of Poles

Therefore, for *k* frequency points *AX*=*b* where,



Computation of zeros of $\sigma(s)$: **Real Case**

With, $X = [\tilde{c}_1 \dots \tilde{c}_n d e c_1 \dots c_N]^T$ known from AX=b

Refined poles for the next iteration can be obtained in terms of the zeros of $\sigma(s)$

Zeros \bar{z}_n of $\sigma(s)$ are the eigenvalues of the matrix

 $H = A - B C^{T}$



Computation of Poles: Multiport Case

$$Y(s) = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1P} \\ Y_{21} & Y_{22} & \dots & Y_{2P} \\ \dots & \dots & \dots & \dots \\ Y_{P1} & Y_{P2} & \dots & Y_{PP} \end{bmatrix};$$

$$F_{p1} = \begin{bmatrix} \sum_{n=1}^{N} \frac{\tilde{c}_{n}^{1,1}}{s - \bar{p}_{n}} + d^{1,1} & \sum_{n=1}^{N} \frac{\tilde{c}_{n}^{1,2}}{s - \bar{p}_{n}} + d^{1,2} & \dots & \sum_{n=1}^{N} \frac{\tilde{c}_{n}^{1,\bar{p}}}{s - \bar{p}_{n}} + d^{1,P} \\ \sum_{n=1}^{N} \frac{\tilde{c}_{n}^{2,1}}{s - \bar{p}_{n}} + d^{2,1} & \sum_{n=1}^{N} \frac{\tilde{c}_{n}^{2,2}}{s - \bar{p}_{n}} + d^{2,2} & \dots & \sum_{n=1}^{N} \frac{\tilde{c}_{n}^{2,p}}{s - \bar{p}_{n}} + d^{2,P} \\ \dots & \dots & \dots & \dots \\ \sum_{n=1}^{N} \frac{\tilde{c}_{n}^{P,1}}{s - \bar{p}_{n}} + d^{P,1} & \sum_{n=1}^{N} \frac{\tilde{c}_{n}^{P,2}}{s - \bar{p}_{n}} + d^{P,2} & \dots & \sum_{n=1}^{N} \frac{\tilde{c}_{n}^{p,p}}{s - \bar{p}_{n}} + d^{P,P} \end{bmatrix}$$

→ A. Chinea and S. Grivet-Talocia, "On the Parallelization of Vector Fitting Algorithms", IEEE Trans. on Components, Packaging and Manufacturing Technology, vol. 1, no. 11, November 2011.

Review of Parallel VF for Multi-CPU Environment



Splitting Strategies

None Splitting	All Splitting
Uses only one Scaling Function for the Entire S matrix	Uses individual Scaling Function for each S- Element
Leads to a common pole-set	Leads to individual pole sets for each S-element
→ CPU1: $A_1 = [Q_1, R_1]$ CPU2: $A_2 = [Q_2, R_2]$	→ CPU1: $A_1 = [Q_1, R_1] \rightarrow \hat{c}_1$ CPU2: $A_2 = [Q_2, R_2] \rightarrow \hat{c}_2$
: CPUT: $\boldsymbol{A}_T = [\boldsymbol{Q}_T, \boldsymbol{R}_T]$	CPUT: $A_T = [Q_T, R_T] \rightarrow \hat{c}_T$
Compute \hat{c} in a single CPU	

GVF: GPU Based Vector Fitting

- Emerging Computing Platform of GPUs
- Thousands of Cores
- Exploit massive parallelization potential

**S. Ganeshan, N. Kumar and R. Achar and W. Lee, "GVF: GPU based Vector Fitting for Modelling of Multiport Tabulated Data Networks", IEEE CPMT, pp. 1375-1387, Aug. 2020.

GVF: GPU based parallel Vector Fitting -None Splitting Strategy

Bulk Transfer from CPU to GPU: S_l , g, α_l

GPU



Formulate and transfer to GPU: $\widehat{\Phi}$



Formulate: $A_1 \cdots A_{p^2}$



Perform QR: $R_1 \cdots R_{P^2}$ (MAGMA)



Extract: $R_{l_{N\times N}}^{22}$: $R_1^{22} \cdots R_{P^2}^{22}$



Collect $R_{l_{N\times N}}^{22}$ for each A_l for evaluation of residues of the scaling function



Example 1 – 60 ports, 70 poles, 1000 Frequency Samples

Size of $\widehat{\Phi}$ = (1000 x 70); $A_l(P^2 = 3600) \rightarrow$ (2000 x 143)

Cost of individual steps in GVF	Time (msec)	Time (msec)
(Using MAGMA library)	(Approach (a))	(Approach (b))
Transfer of S, g, α from CPU to GPU	20.19	20.19
Formulation of $\widehat{\Phi}$ in CPU and transfer to GPU	1.97	1.97
Formulation of <i>A</i> in GPU (<i>customized</i>)	0.04	0.04
QR factorization in GPU (MAGMA)	543.09	543.09
R_l^{22} extraction in GPU (<i>customized</i>)	0.02	0.02
Transfer back R_l^{22} from GPU to CPU	56.00	-
Residue \hat{c} calculation in CPU (LAPACK)	460.02	-
Residue \hat{c} calculation in GPU (MAGMA)	-	562.50
Transfer back \hat{c} from GPU to CPU	-	0.03
Zeros computation in CPU	3.60	3.60
Total Time Taken	1084.9	1131.44

Example 1 – 60 ports, 70 poles, 1000 Frequency Samples: None Splitting Strategy



GVF: All Splitting Strategy



Example 1 – 60 ports, 70 poles, 1000 Frequency Samples: All Splitting Strategy



GVF: GPU based Vector fitting algorithm Example case: 120 ports, 80 poles, 1000 Frequency Samples All Splitting Strategy

	PVF [34] (multi CPU)		Proposed GVF time	
Example	# CPU	Time	using MAGMA library	Speedup
	cores	(sec)	(sec)	
	used			
#ports = 120 #poles = 80 #fpoints = 1000 #size of $\widehat{\Phi}$ =1000 × 80 Total # of Φ_l =14400	1	193.90	42.86	4.52
	2	95.45	21.21	4.50
	4	48.40	11.10	4.36
	6	36.80	8.72	4.22
	8	25.03	6.03	4.15
	12	17.71	4.43	4.00
	16	14.42	3.81	3.78
	20	12.23	3.69	3.31

Example 2 – 120 ports, 80 poles, 1000 Frequency Samples: All Splitting Strategy



Conclusions

Advanced Interconnect Modeling and Simulation methods still emerging:

- to meet the fundamental challenges such as passivity
- to be more efficient/accurate
- to meet the high-speed designer's dream of seamless analysis platform for mixed circuit/EM/RF design.



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