Modelaje y extracción de parámetros de celdas y paneles solares

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Distinguished Lecture Program

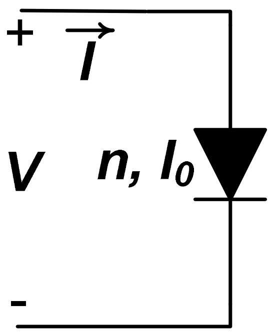
Introducción

- El modelaje y la extracción de parámetros en los paneles solares es un tema muy importante para poder optimizar el uso de la energía solar.
- Revisaremos en forma crítica los métodos mas importantes y las ideas que los motivaron.
- Los métodos serán comparados usando mediciones y simulaciones.

Contenido:

1. Diodo con una exponencial sin resistencia
2. Diodo con una exponencial con resistencia
3. Diodo con una exponencial con resistencias serie y paralela
4. Diodo con doble exponencial
5. Diodo con múltiples exponenciales
6. Celdas solares con una exponencial
7. Métodos de extracción en celdas solares
8. Celdas solares con múltiples exponenciales
9. Resumen

1. Single-exponential model without resistance [SA57]



$$I = I_0\left[\exp\left(\frac{V}{n\,v_{th}}\right)-1\right]$$

neglecting -1 term:

$$\ln(I) = \ln(I_0) + \frac{V}{n\,v_{th}}$$

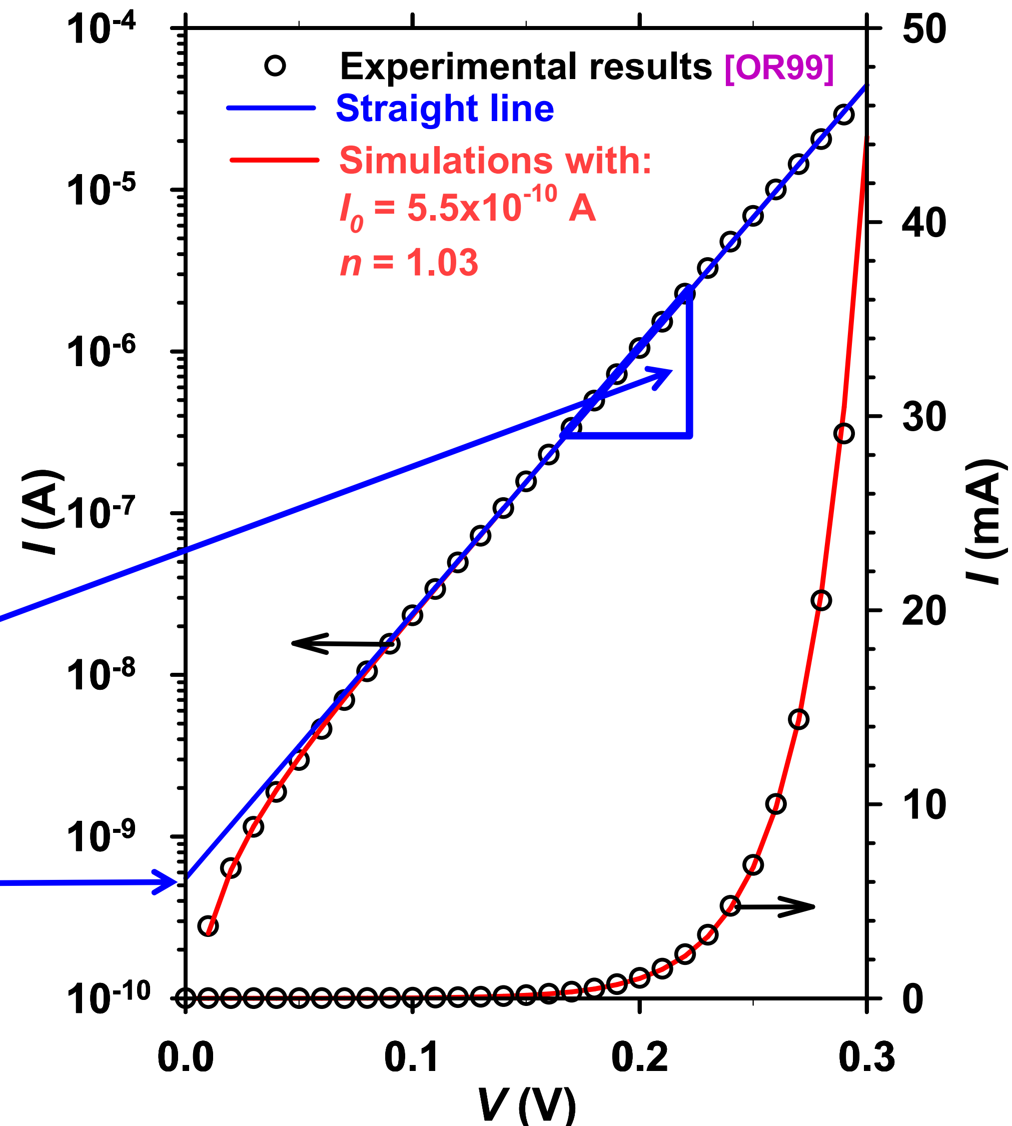
Therefore, ln(I) is an straight line.

The slope yields nv_{th}:

$$n\,v_{th} = \frac{0.3V - 0.1V}{ln\left(2.33x10^{-8}\right) - ln\left(4.18x10^{-5}\right)}$$

$$n = 1.03$$

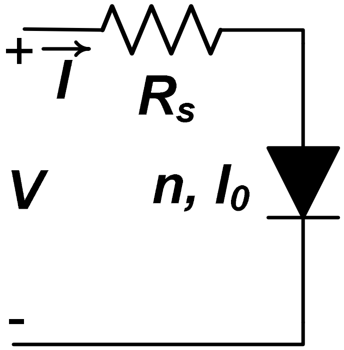
The intercept gives:
I_0 = 5.5x10^{-10} A



[SA57] Sah CT, Noyce RN, Shockley W., "Carrier generation and recombination in P-N junctions and P-N junction characteristics", Proc IRE; 45 (9) : 1228–1243, 1957. http://dx.doi.org/10.1109/JRPROC.1957.278528

[OR99] A. Ortiz-Conde et al "Direct extraction of semiconductor device parameters using lateral optimization method", Solid-State Electronics, vol. 43, pp. 845-848, April 1999. http://dx.doi.org/10.1016/S0038-1101(99)00044-1

2. Single-exponential model with series resistance



Implicit equation

$$I = I_o\left[\exp\left(\frac{V - R_s\, I}{n\, v_{th}}\right) - 1\right]$$

Lateral equation

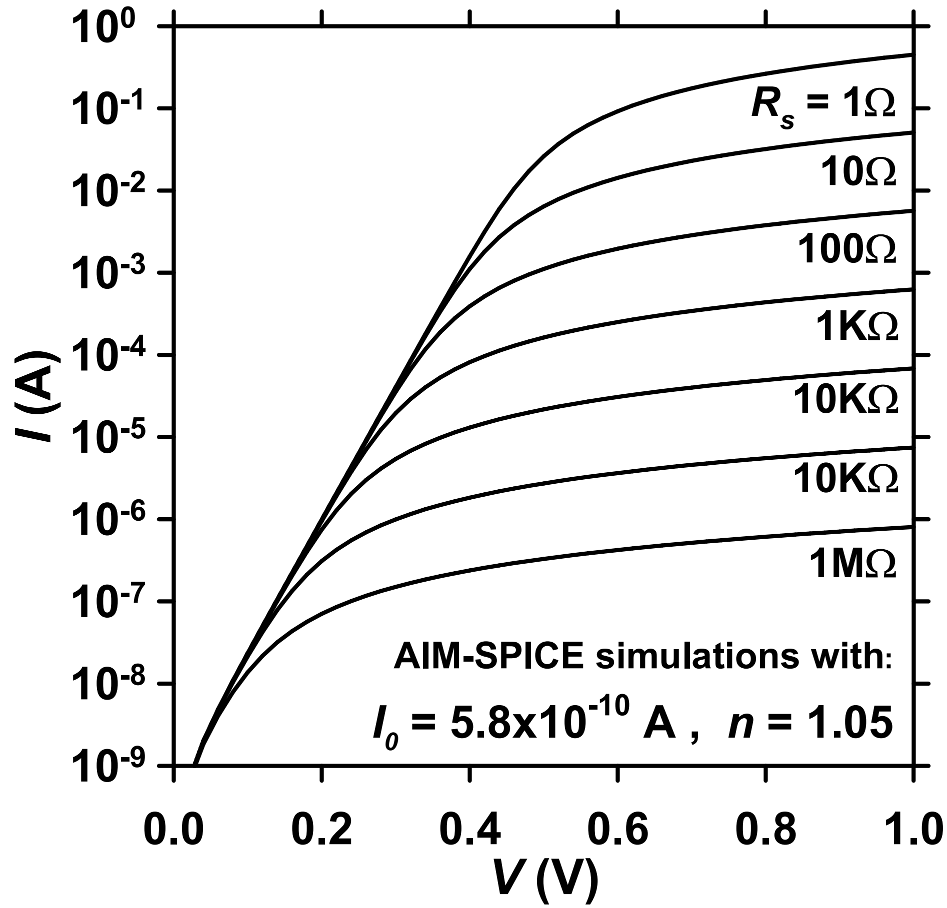
$$V = R_s\, I + n\, v_{th}\, \ln\left(1 + \frac{I}{I_0}\right)$$

Vertical equation

$$I = \frac{nv_{th}}{R_S}\, W\left[exp\left[\frac{V + I_0 R_S}{nv_{th}} + ln\left(\frac{I_0 R_S}{nv_{th}}\right)\right]\right] - I_0$$

$$I = \frac{nv_{th}}{R_S}\, \omega\left[\frac{V + I_0 R_S}{nv_{th}} + ln\left(\frac{I_0 R_S}{nv_{th}}\right)\right] - I_0$$

where *W* and ω are the Lambert and Wright functions respectively.

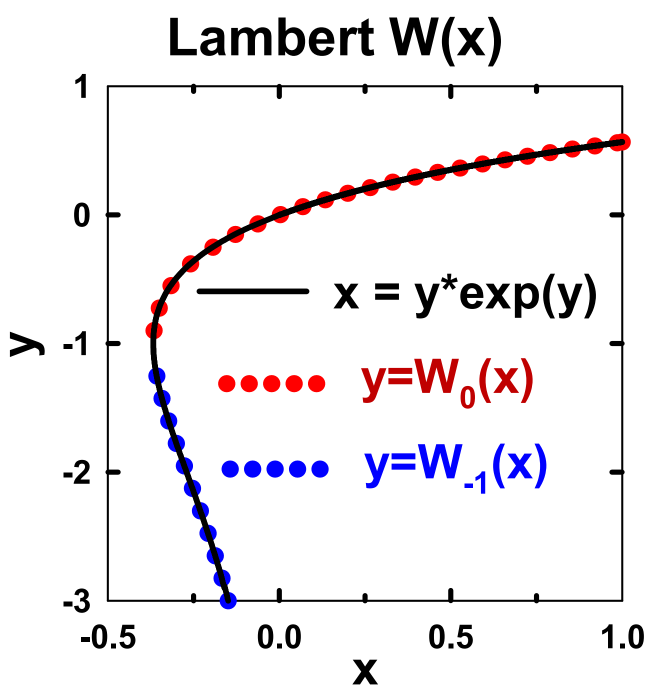


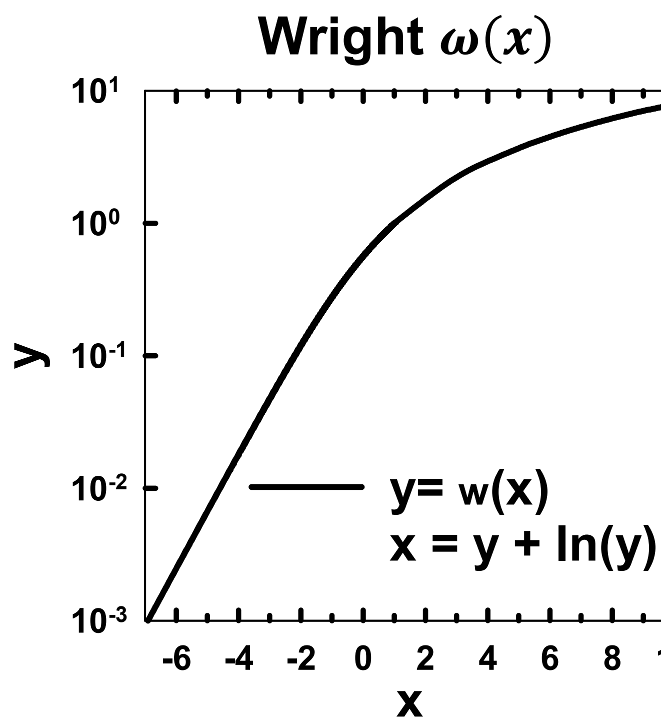
R_s is important for high voltage and it decreases the region where ln(I) is proportional to V.

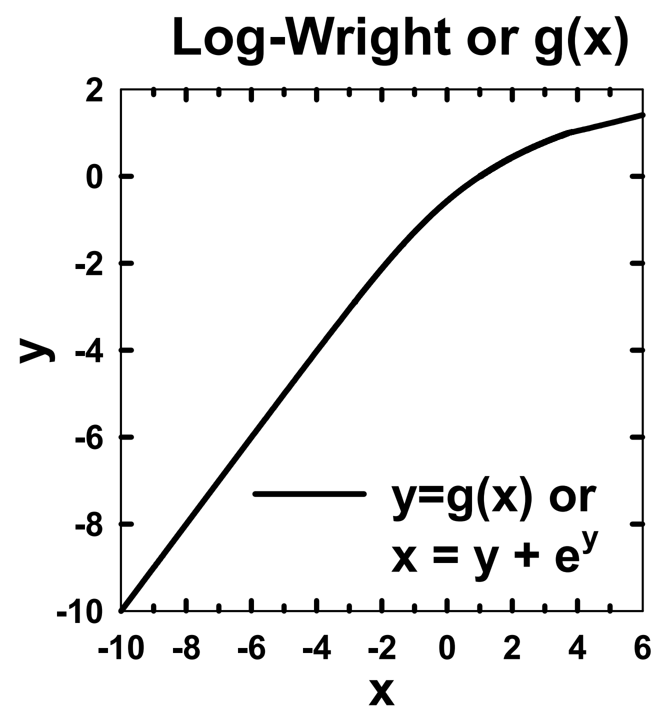
[OR99] A. Ortiz-Conde, Yuanshen Ma, J. Thomson, E. Santos, J. J. Liou, F. J. García Sánchez, M. Lei, J. Finol and P. Layman, "Direct extraction of semiconductor device parameters using lateral optimization method", Solid-State Electronics, vol. 43, pp. 845-848, April 1999. http://dx.doi.org/10.1016/S0038-1101(99)00044-1

Properties of Lambert W, Wright ω, and g functions

Function	Inverse	Identity	Property
Lambert W(x)	x = ye^y	W(x)	multiple valued
Wright $\omega(x)$	x = $y + \ln(y)$	$\omega(x)$=W(e^x)	single valued
Log-Wright or g(x)	x = $y + e^y$	g(x) = ln[W(e^x)]=x-W(e^x)	single valued







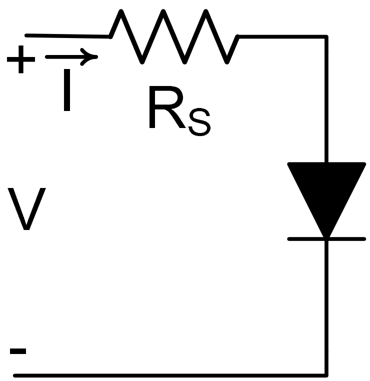
Lambert, Wright ω, and g functions

In MOSFET and solar cells, the solution is $\omega(x)$=W(e^x).
The W(e^x) solution is mathematically correct, but the exponential could yield underflow or overflow [RO15, LA23 CA25].
The Wright ω function [LA23]: "addresses the overflow problem by mathematically modifying the Lambert W function such that intermediate arguments with large numerical magnitudes are avoided".
The Log-Wright function, $\mathrm{g}(\mathrm{x})$ = ln$[$W$(e^x)]$ can be easily implemented by using the Halleys' method [RO15, CA25]:

$$y_{n+1} = y_n - \frac{2(y_n + e^{y_n} - x)(1 + e^{y_n})}{2(1 + e^{y_n})^2 - (y_n + e^{y_n} - x)e^{y_n}}$$

[RO15] K. Roberts, "A Robust Approximation to a Lambert-Type Function", 2015. https://arxiv.org/abs/1504.01964
[LA23] Lankireddy, P. et al., "Solar Cells, Lambert W and the LogWright Functions", Int. Conf. on Electrical, Computer and Energy Techn., ICECET 2023. https://doi.org/10.1109/ICECET58911.2023.10389599
[CA25] M. Calasan, "Iterative methods for solving g-functions: a review, comparative evaluation, and application in the solar cell domain", Journal of Computational Electronics, 24:58, 2025. https://doi.org/10.1007/s10825-025-02298-2

2.1 Vertical optimization method using the implicit equation



Minimizing the errors on the current which is numerically solve from the implicit equation:

$$I = I_o \left[\exp\left(\frac{V - R_S\, I}{n\, v_{th}} \right) - 1 \right]$$

It is not computationally efficient.

2.2 Vertical optimization method using the vertical equation [HR06]

Minimizing the errors on the current which is evaluated using:

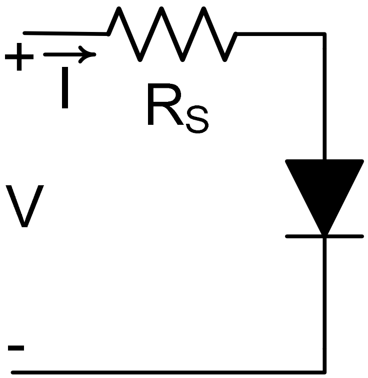
$$I = \frac{nv_{th}}{R_S} W\left[exp\left[\frac{V + I_0 R_S}{nv_{th}} + ln\left(\frac{I_0 R_S}{nv_{th}} \right) \right] \right] - I_0$$

$$I = \frac{nv_{th}}{R_S}\ \omega\left[\frac{V + I_0 R_S}{nv_{th}} + ln\left(\frac{I_0 R_S}{nv_{th}} \right) \right] - I_0$$

It is computationally efficient but it requires using the Lambert or Wright functions.

[HR06] Hruska, P., Chobola, Z., Grmela, L., “Diode I-U curve fitting with Lambert W function”, 25th International Conference on Microelectronics (MIEL), pp. 501-504., 2006. http://dx.doi.org/10.1109/ICMEL.2006.1651003 .

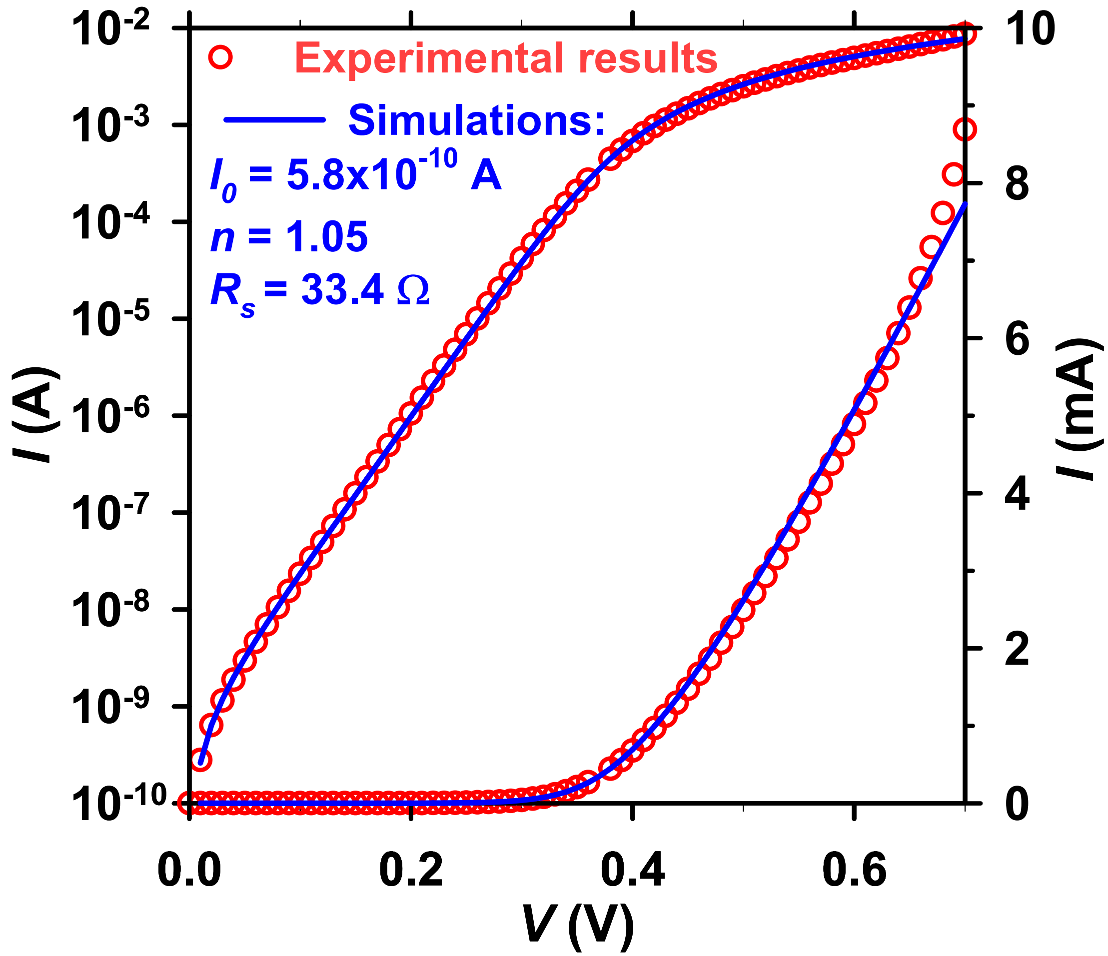
2.3 Lateral optimization method using the lateral equation [BE87;OR99]



Minimizing the errors on the voltage, instead of the current:

$$V = R_S\, I + n\, v_{th} \ln\left(1 + \frac{I}{I_0}\right)$$

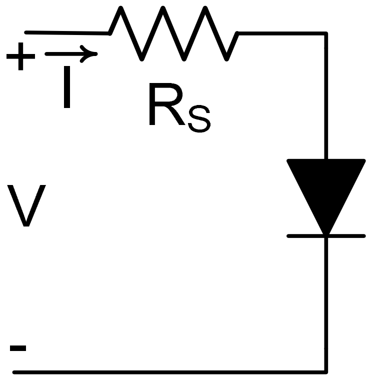
It is computationally efficient and it requires using a simple log function.



[BE87] Bennett, R.J., "Interpretation of forward bias behavior of schottky barriers", IEEE Transactions on Electron Devices ED-34 (4) , pp. 935-937, 1987. http://dx.doi.org/10.1109/T-ED.1987.23020

[OR99] A. Ortiz-Conde, Yuanshen Ma, J. Thomson, E. Santos, J. J. Liou, F. J. García Sánchez, M. Lei, J. Finol and P. Layman, "Direct extraction of semiconductor device parameters using lateral optimization method", Solid-State Electronics, vol. 43, pp. 845-848, April 1999. http://dx.doi.org/10.1016/S0038-1101(99)00044-1

2.4 Integration method to extract n and R_s for diodes [OR93,95]



Using $V(I)$ and integration by part yields

$$\int_0^V I\,dV = V\,I - \int_0^I V\,dI = \frac{R_s}{2} I^2 + n\,v_{th}\,I - I_0\,V$$

$$V = R_s\,I + n\,v_{th}\ln\left(1 + \frac{I}{I_0}\right)$$

Assuming $I >> I_0$: $$\int_0^V I\,dV \approx \frac{R_s}{2} I^2 + n\,v_{th}\,I$$

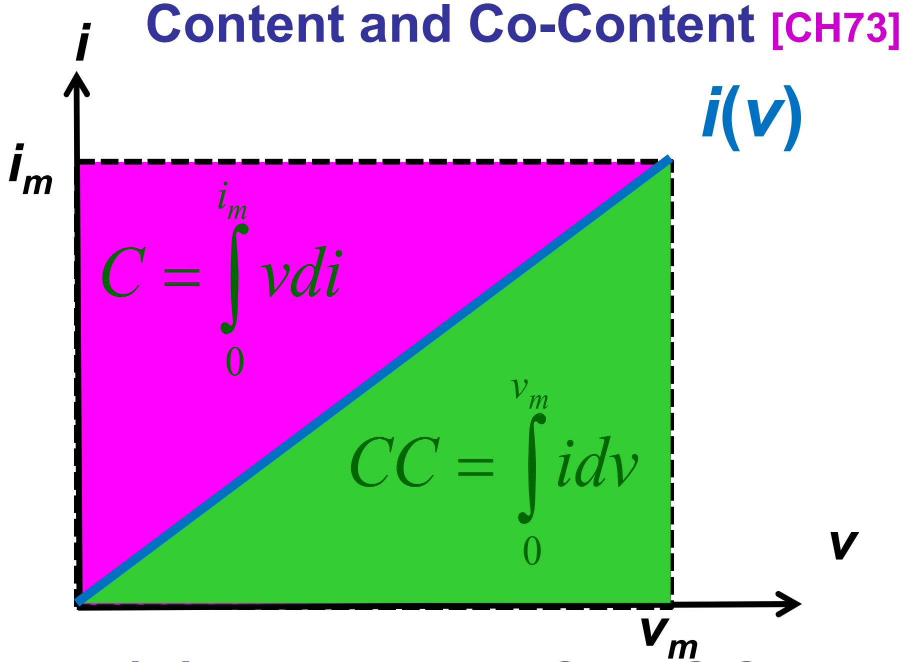
This is a much simpler problem of fitting an explicit algebraic quadratic equation:

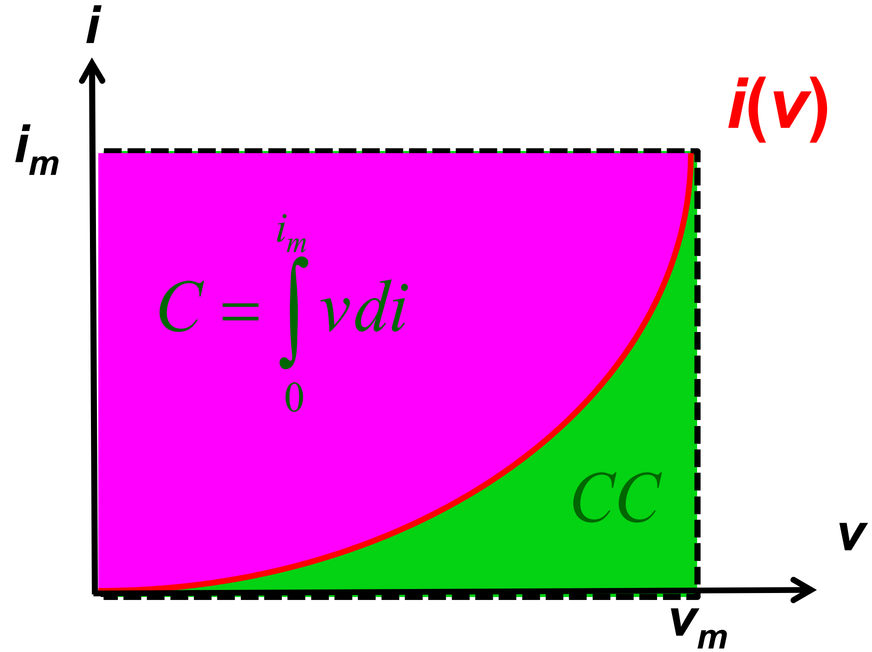
[OR93] A. Ortiz-Conde, F.J. García Sánchez, P.E. Schmidt, and R.J. Laurence, Jr., "Extraction of diode parameters from the integration of the forward current", Proc. of the Second International Semiconductor Device Research Symposium (Charlottesville, Virginia), vol. 2, pp. 531-534, Dec. 1993.

[OR95] A. Ortiz-Conde, F.J. García Sánchez, J.J. Liou, J. Andrian, R.J. Laurence, and P.E. Schmidt, "A generalized model for a two-terminal device and its application to parameters extraction", Solid-State Electronics, vol. 38, pp. 265-266, Jan. 1995. http://dx.doi.org/10.1016/0038-1101(94)00141-2

2.4.1 The Integral Difference Function and the G method [GA95,96]

Content and Co-Content [CH73]





When *i*(*v*) is linear: *C* = *CC*. **When *i*(*v*) is non-linear: *C* ≠ *CC*.**

$$D \equiv C - CC = \int_0^{i_m} v di - \int_0^{v_m} i dv = iv - 2\int_0^{v_m} i dv = 2\int_0^{i_m} v di - iv$$

[GA95] F. J. García Sánchez, A. Ortiz-Conde, G. Mercato, J. J. Liou, and L. Recht, "Eliminating parasitic resistances in parameter extraction of semiconductor device models", Int. Caracas Conf. on Cir. Dev. and Sys., pp. 298-302, Dec. 1995. http://dx.doi.org/10.1109/ICCDCS.1995.499164

[GA96] F.J. García Sánchez, A. Ortiz-Conde, and J.J. Liou, "A parasitic series resistance-independent method for device-model parameter extraction", IEE Proc. Cir. Dev. and Sys., vol. 143, pp. 68-70, Feb. 1996. http://dx.doi.org/10.1049/ip-cds:19960159

[CH73] L. Chua, “Stationary principles and potential functions for nonlinear networks,” J. Franklin Inst., 296, 91-114, 1973. http://dx.doi.org/10.1016/0016-0032(73)90077-X

2.4.1 The Integral Difference Function and the G method

Eliminating series resistance

1° **Calculate D:**

$$D = IV - 2\int_0^V I\,dV = D_J + \cancelto{0}{D_{R_S}}$$

Substituting Shockley's equation for $I >> I_0$:

$$D = D_J \approx I\, n v_{th}\left[\ln\left(I/I_0\right) - 2\right]$$

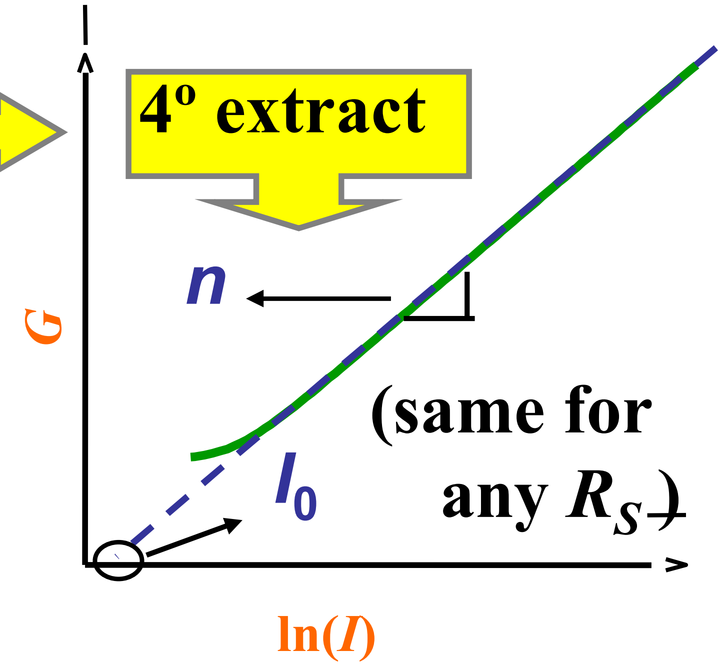
New function: G

2°

$$G \equiv D/I$$

$$\approx \underbrace{n\, v_{th}}_{\text{slope}} \left\{\ln(I) - \underbrace{\left[\ln(I_0) + 2\right]}_{\text{intersection}}\right\}$$

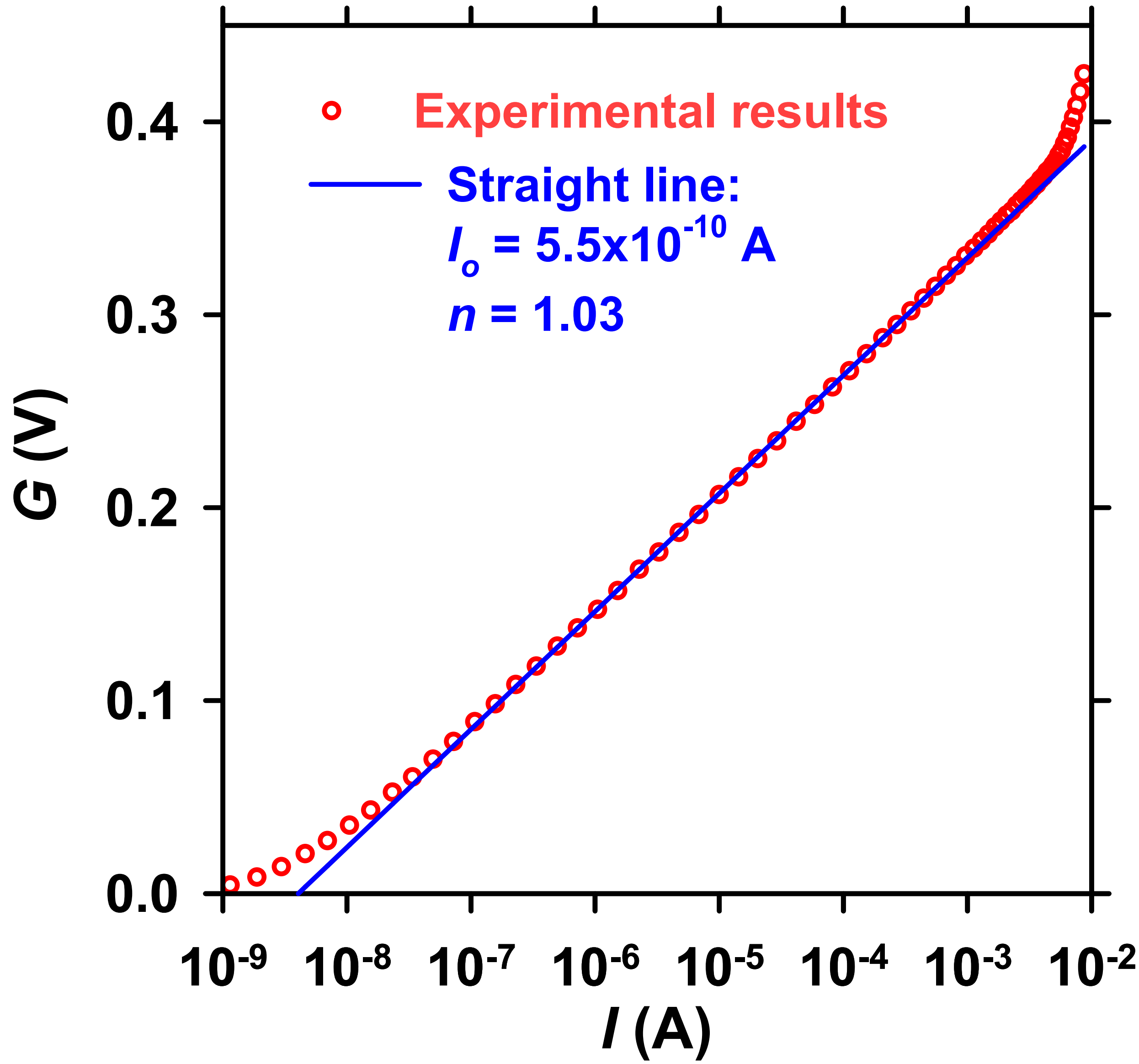
3° plot G



2.4.1 The Integral Difference Function and the G method

$$G \approx n\,v_{th}\left\{\ln(I) - \left[\ln(I_0) + 2\right]\right\}$$

The extracted values of
n=1.03 and I_0=5.5x10^{-10} A
are very close to
previously obtained by
lateral optimization:
n=1.05 and I_0=5.8x10^{-10} A



2.4.2 The ΔG method [OR18]

Using previous result: $$G \equiv D/I \equiv V - \frac{2}{I}\int_0^V I\,dV \approx n\,v_{th}\left[\ln\left(I/I_0\right) - 2\right]$$

Defining: $$\Delta G(V,I) \equiv G(V,I) - G(V_R,I_R) = n\,v_{th}\ln\left(I/I_R\right)$$

where (V_R, I_R) is a selected reference point

Parameter *n* is obtained: $$n = \frac{\Delta G(V,I)}{v_{th}\ln\left(I/I_R\right)}$$

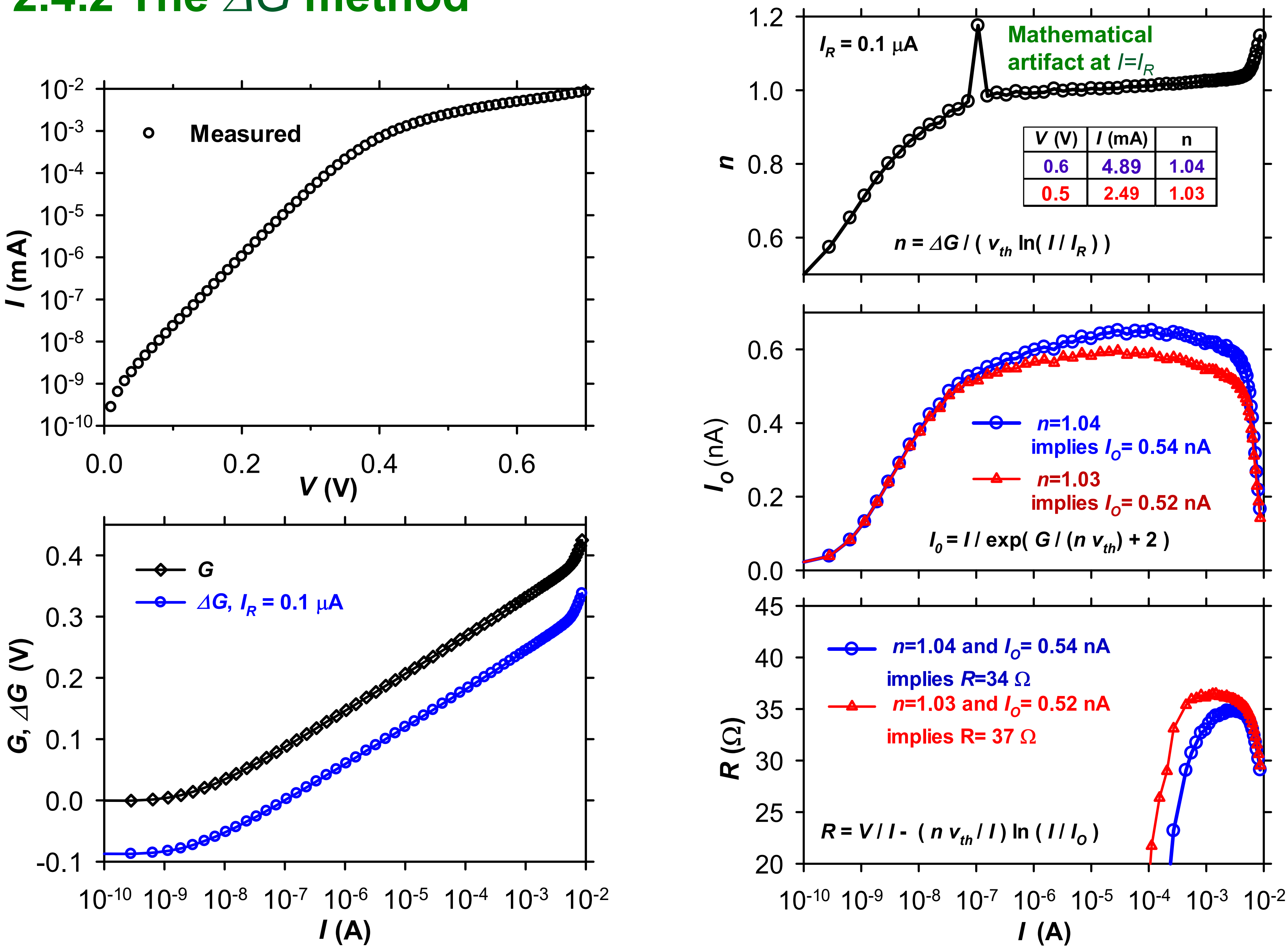
Then $$I_0 = \frac{I}{\exp\left(\dfrac{G}{n\,v_{th}} + 2\right)}$$

Finally $$R = \frac{V}{I} - \frac{n\,v_{th}}{I}\ln\left(\frac{I}{I_0} + 1\right) \approx \frac{V}{I} - \frac{n\,v_{th}}{I}\ln\left(\frac{I}{I_0}\right)$$

This method isolates the effects of each of the parameters.

[OR18] A. Ortiz-Conde and F. J. García Sánchez, "A new approach to the extraction of single exponential diode model parameters ", accepted in Solid-State Electronics, Feb. 2018. https://doi.org/10.1016/j.sse.2018.02.013

2.4.2 The ΔG method



2.5 A METHOD BASED IN DIFFERENTIATION [OR18]

Performing the first and second derivative of *V* with respect to *I* :

$$V = R\,I + n\,v_{th}\ln\left(1+\frac{I}{I_0}\right) \qquad \frac{dV}{dI} = R + \frac{n\,v_{th}}{I+I_0} \qquad \frac{d^2V}{dI^2} = -\frac{n\,v_{th}}{(I+I_0)^2}$$

Note that *R* has been eliminated in last equation. Expressing the derivatives of *V* with respect to *I* in term of the derivatives of *I* with respect to *V*:

$$\frac{dV}{dI} = \left(\frac{dI}{dV}\right)^{-1} \qquad \frac{d^2V}{dI^2} = -\frac{\dfrac{d^2I}{dV^2}}{\left(\dfrac{dI}{dV}\right)^3}$$

Assuming $I \gg I_0$ and solving for parameter *n* :

$$n \approx -\frac{I^2\,\dfrac{d^2I}{dV^2}}{v_{th}\left(\dfrac{dI}{dV}\right)^3}$$

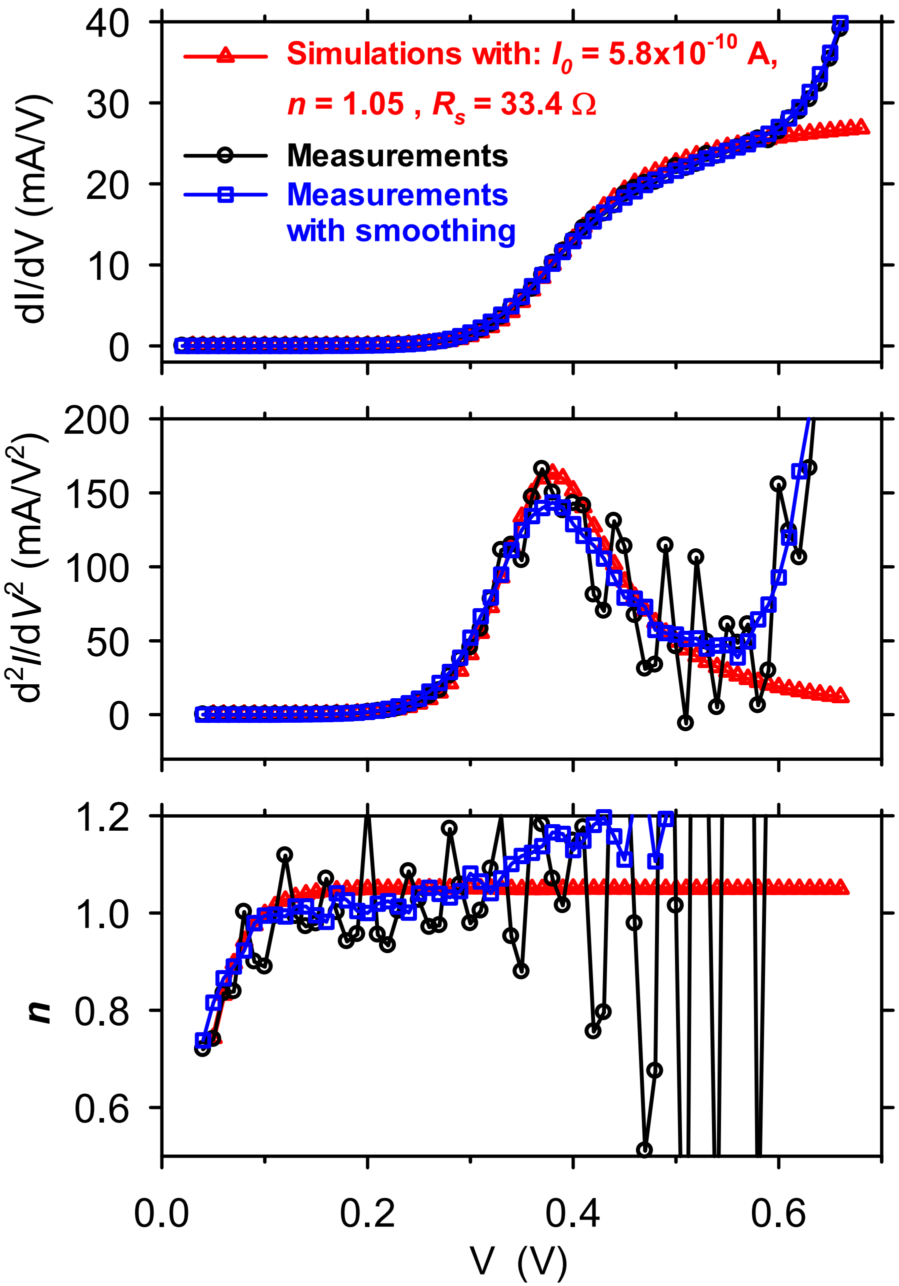
Solving for the other parameters:

$$R = \frac{dV}{dI} - \frac{n\,v_{th}}{I} \qquad I_0 = I\exp\left(\frac{R\,I - V}{n\,v_{th}}\right)$$

This method isolates the effects of each of the parameters.

[OR18] A. Ortiz-Conde and F. J. García Sánchez, "A new approach to the extraction of single exponential diode model parameters ", accepted in Solid-State Electronics, Feb. 2018. https://doi.org/10.1016/j.sse.2018.02.013

2.5 A METHOD BASED IN DIFFERENTIATION

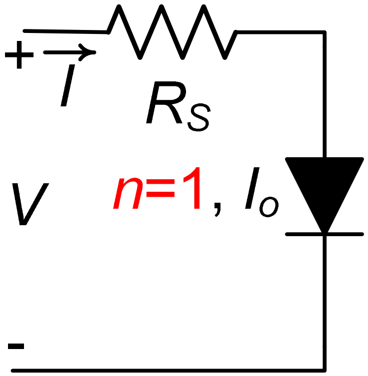


The differentiation method is successful with simulated data and it fails with measurements.

The differentiation method is simpler and can be used when the noise in the measured data is very small.

On the other hand, the integration method is more sophisticated and more immune to the noise in the measured data.

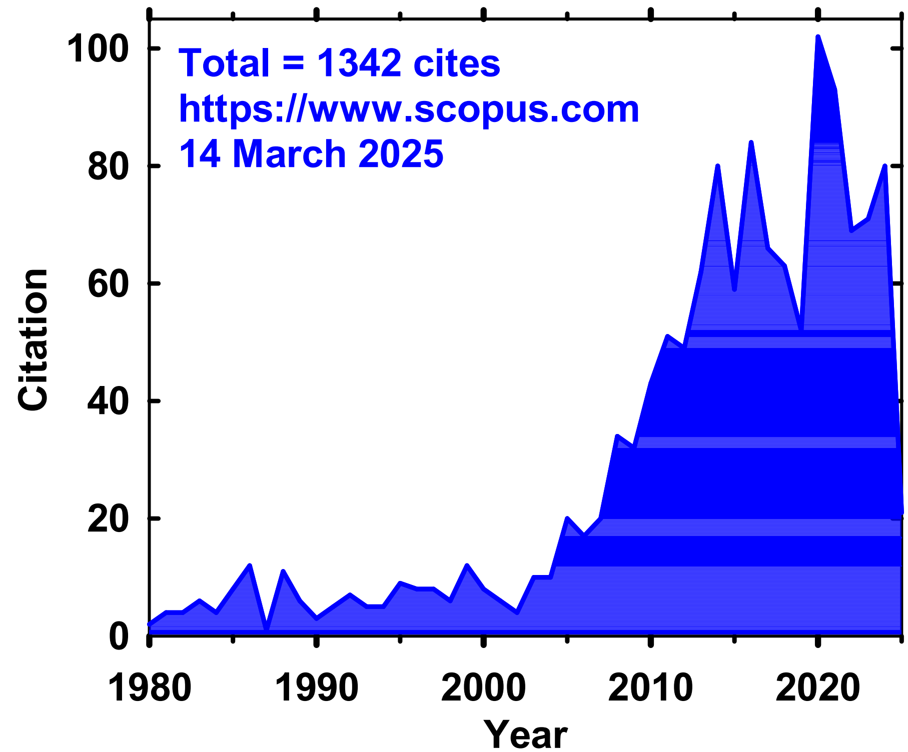
2.6 Norde's method [NO79]



This method contains clever mathematical ideas and it was developed for Schottky diodes with n=1. The following notation is adapted to p-n junctions.

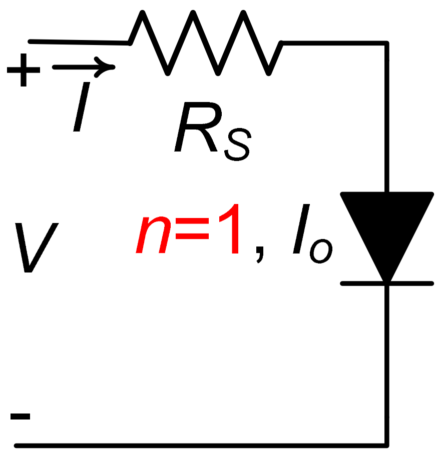
Although it is a 46-year-old method, it is still very popular.

Last year, it was cited 80 times.



[NO79] Norde, H., "A modified forward I-V plot for Schottky diodes with high series resistance", Journal of Applied Physics50 (7) , pp. 5052-5053, 1979. http://dx.doi.org/10.1063/1.325607

2.6 Norde's method



$$I = I_0 \left[\exp\left(\frac{V - I\,R_s}{v_{th}} \right) - 1 \right] \quad (1)$$

$$V = R_s\,I + v_{th} \ln\left(\frac{I}{I_0} + 1 \right) \quad (2)$$

Norde defined the following function, and we call it by his name:

$$Norde = \frac{V}{2} - v_{th} \ln\left(\frac{I}{I_x} \right) \quad (3)$$

where I_x is an arbitrary value of current.
Norde's function presents a minimum value at V_{min}, I_{min} which is independent on the selected value of I_x.

2.6 Norde's method

The minimum value is obtained by deriving (3) and equating it to zero:

$$\frac{d\,Norde}{d\,V} = \frac{1}{2} - \frac{v_{th}}{I}\frac{d\,I}{d\,V} \qquad (4)$$

Note that the location of the minimum is independent on the selected value of I_x

Using (2):

$$\frac{d\,V}{d\,I} = R_s + \frac{v_{th}}{I} \qquad (5)$$

Substituting (5) into (4):

$$\frac{d\,Norde}{d\,V} = \frac{1}{2} - \frac{v_{th}}{I\left(R_s + \frac{v_{th}}{I}\right)} = \frac{1}{2} - \frac{v_{th}}{IR_s + v_{th}} = 0 \qquad (6)$$

$$\Downarrow$$

$$\boxed{R_s = \frac{v_{th}}{I_{\min}} \qquad (7)}$$

where I_{min} is the value of the current at the minimum of Norde's function.

2.6 Norde's method

Evaluating I_0 from (1):

$$\boxed{I_0 = \frac{I_{\min}}{\left[\exp\left(\dfrac{V_{\min} - I_{\min} R_s}{v_{th}}\right) - 1\right]} \qquad (8)}$$

where V_{min} is the value of the voltage at the minimum of Norde's function.

The disadvantages of Norde's method are:
1) The ideality factor is assumed to be unity.
2) The extracted parameters are based on a few data points near the minimum of Norde's function.

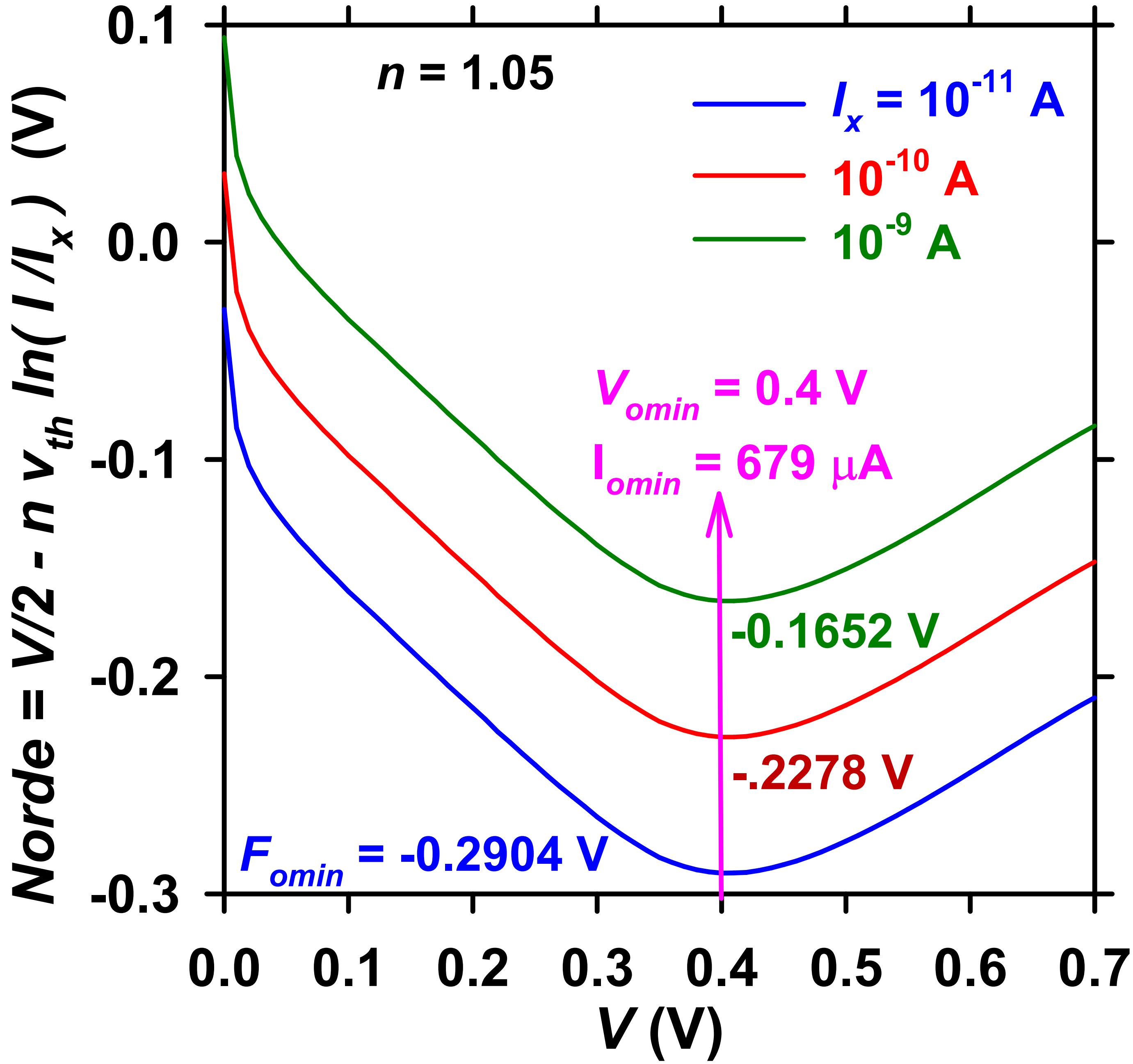
We can say that this is a clever transitional method which extract the parameters at a value where the resistance and the diode are important.

2.6 Norde's method

We will test Norde's method using the same experimental data [OR99], which was previously modeled with [OR99]:
I_0 = 580 pA
n = 1.05
R_s = 33.4 Ω

Since n = 1.05 and it should be unity for the present method, we fix V_{th}= 1.05x0.259 V

For the three selected value of I_x, the extracted values are:
R_s = 40 Ω
I_0 = 756 pA

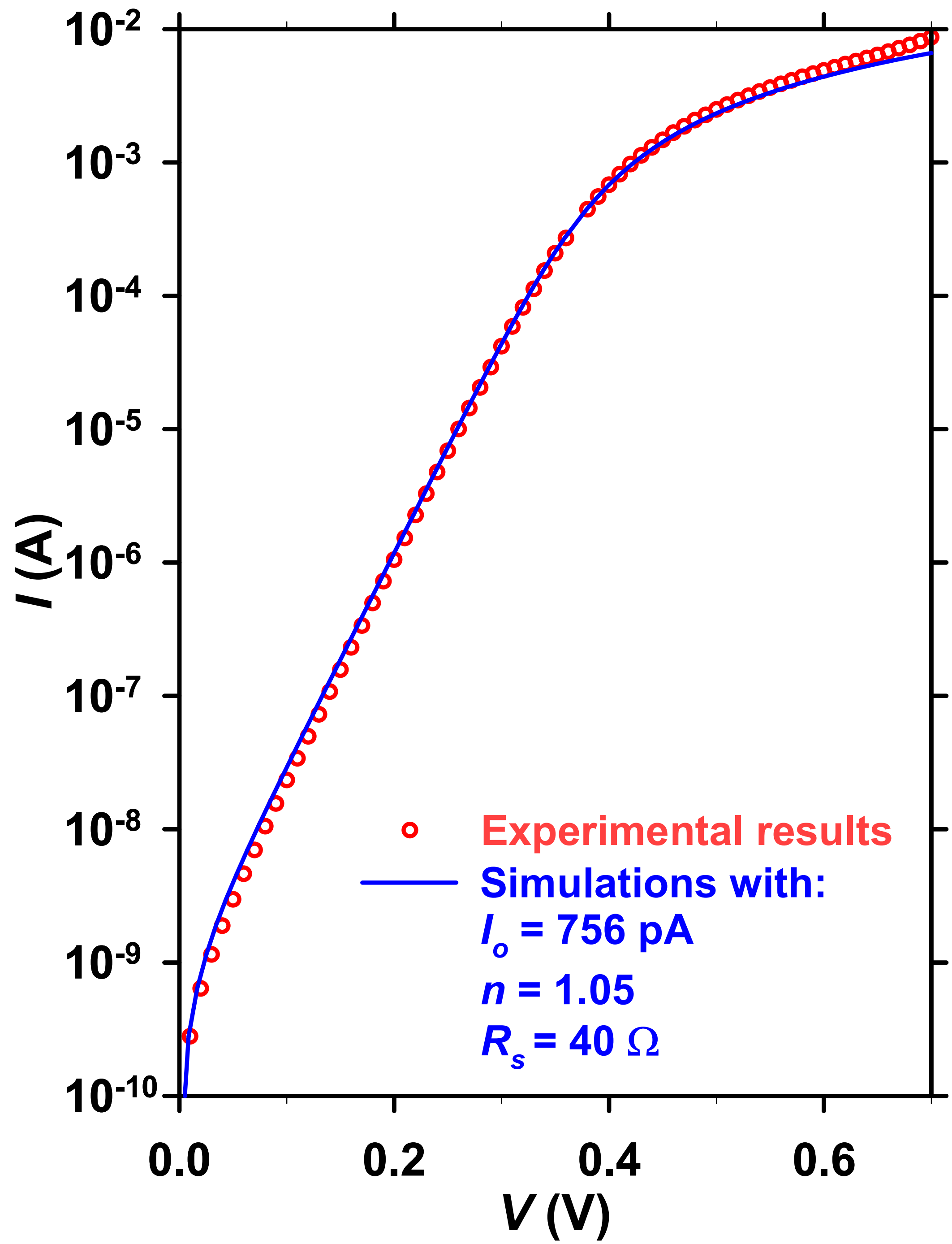


2.6 Norde's method

The simulation, using the parameters extracted with Norde's method, agrees very well with experimental data for values close to V_{min} = 0.4 V.

The limitation of n=1 has been removed by various authors

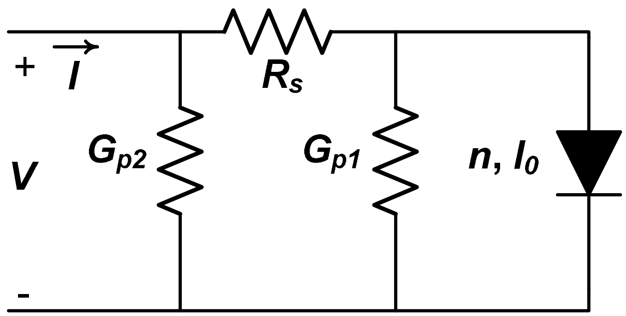
[LI84;BO86]



[LI84] Lien, C.-D. ; So, F.C.T. ; Nicolet, M.A., "An improved forward I-V method for nonideal Schottky diodes with high series resistance", IEEE Trans. Electron Dev., 31, p. 1502-1503, Oct. 1984. http://dx.doi.org/10.1109/T-ED.1984.21739

[BO86] Bohlin, K.E., "Generalized Norde plot including determination of the ideality factor", J. Appl. Phys., 60, p. 1223-1224, 1986. http://dx.doi.org/10.1063/1.337372

3. Single-exponential model with series and parallel resistances



Original implicit equation:

$$I = I_O\left(\exp\left(\frac{V\left(1+R_s\,G_{p2}\right)-I\,R_s}{n\,v_{th}}\right)-1\right) + \left(V - I\,R_s\right)G_{p1} + V\,G_{p2}\left(1+R_s\,G_{p1}\right)$$

Explicit solutions:

$$I = \frac{nv_{th}}{R_s} W\left\{\frac{I_0 R_s}{nv_{th}\left(1+R_s G_{p1}\right)}\exp\left[\frac{\left(V+I_0 R_s\right)}{nv_{th}\left(1+R_s\,G_{p1}\right)}\right]\right\} + \left(\frac{V\,G_{p1}-I_0}{1+R_s\,G_{p1}}\right) + V\,G_{p2}$$

$$V = -nv_{th}\,d_2\,W\left\{\frac{I_0\,R_{12}}{nv_{th}\,d_2}\exp\left[\frac{\left(I+\frac{I_0}{d_2}\right)R_{12}}{nv_{th}}\right]\right\} + I\,d_2\left[R_s + R_{12}\right] + I_0\,R_{12}$$

where:
W is the principal branch of the **Lambert Function.**

$$d_1 \equiv 1\big/\left(1+R_s\,G_{p1}\right) \qquad d_2 \equiv 1\big/\left(1+R_s\,G_{p2}\right) \qquad R_{12} \equiv 1\big/\left(G_{p1}+G_{p2}+G_{p1}G_{p2}\,R_s\right)$$

G_{p1} and G_{p2} are two parallel parasitic conductances, one at the junction and at the other at the periphery.

[OR00] A. Ortiz-Conde, F.J. García Sánchez, J. Muci, "Exact analytical solutions of the forward non-ideal diode equation with series and shunt parasitic resistances", Solid-State Electronics, vol. 44, pp. 1861-1864, Oct. 2000. http://dx.doi.org/10.1016/S0038-1101(00)00132-5

3.1 Function *D* vs *I* and *V* [OR05]

Integrating

$$V = -nv_{th}\, d_2\, W\left\{ \frac{I_0\, R_{12}}{nv_{th}\, d_2} \exp\left[\frac{\left(I + \frac{I_0}{d_2} \right) R_{12}}{nv_{th}} \right] \right\} + I\, d_2 \left[R_S + R_{12} \right] + I_0\, R_{12}$$

with respect to *I*, and substituting into:

$$D = 2\int_0^I V dI - IV$$

permits to calculate function *D* in the form of a convenient purely algebraic expression:

$$D(I,V) = D_{V1}V + D_{I1}I + D_{V1I1}VI + D_{V2}V^2 + D_{I2}I^2$$

[OR05] A. Ortiz-Conde and F. J. García Sánchez, "Extraction of non-ideal junction model parameters from the explicit analytic solutions of its I–V characteristics", Solid-State Electronics, Vol.49, pp. 465-472, March 2005. http://dx.doi.org/10.1016/j.sse.2004.12.001

3.1 Function *D* vs *I* and *V*

where the coefficients are:

$$\mathbf{D_{I1}} = -2R_s I_0 - 2n v_{th} \left(1 + G_{p1} R_s\right)$$

$$\mathbf{D_{V1}} = 2R_s n v_{th} G_{p1} G_{p2} + 2I_0 (R_s G_{p2} + 1) + 2n v_{th} (G_{p1} + G_{p2})$$

$$\mathbf{D_{I1V1}} = 1 + 2R_S \left(G_{P1} + G_{P2}\right) + 2R_S{}^2 G_{P1} G_{P2}$$

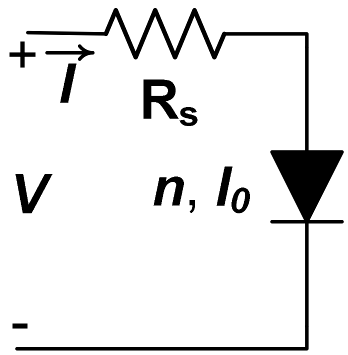
$$\mathbf{D_{I2}} = -R_s \left(1 + G_{p1} R_s\right)$$

$$\mathbf{D_{V2}} = -G_{p2} - G_{p1} - 2R_s G_{p1} G_{p2} - R_s G_{p2}^2 - R_s^2 G_{p1} G_{p2}^2$$

Unfortunately, the five coefficients are not all independent:

$$\boxed{D_{I1V1}^2 = 1 + 4\, D_{I2}\, D_{V2}}$$

3.1 Function *D* vs *I* and *V* – *Particular cases*



$$\mathbf{D_{I1}} = -2R_S I_0 - 2n\, v_{th}$$

$$\mathbf{D_{V1}} = 2I_0 \qquad \mathbf{D_{I1V1}} = 1$$

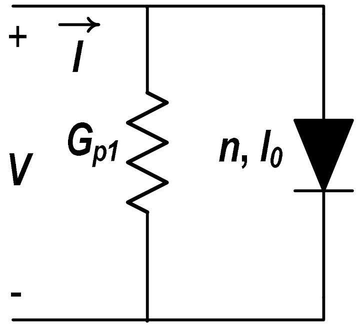
$$\mathbf{D_{I2}} = -R_S \qquad \mathbf{D_{V2}} = 0$$

Solving R_s, n and I_o :

$$R_S = -\mathbf{D_{I2}} \qquad I_0 = \frac{\mathbf{D_{V1}}}{2}$$

$$n = -\frac{\mathbf{D_{I1}} + 2R_s I_0}{2\, v_{th}}$$

3.1 Function *D* vs *I* and *V* – *Particular cases*



$$\mathbf{D_{I1}} = -2nv_{th} \qquad \mathbf{D_{I1V1}} = 1$$

$$\mathbf{D_{V1}} = 2I_0 + 2nv_{th}G_{p1}$$

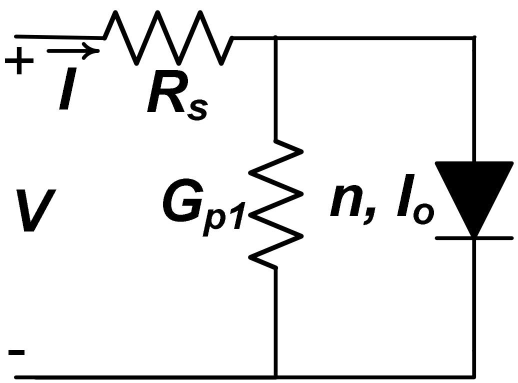
$$\mathbf{D_{V2}} = -G_{p1} \qquad \mathbf{D_{I2}} = 0$$

Solving G_{p1}, *n* and I_o :

$$G_{P1} = -\mathbf{D_{V2}} \qquad n = -\frac{\mathbf{D_{I1}}}{2v_{th}}$$

$$I_0 = \frac{\mathbf{D_{V1}} - 2nv_{th}G_{p1}}{2}$$

3.1 Function *D* vs *I* and *V* – *Particular cases*



$$\mathbf{D_{I1}} = -2R_s I_0 - 2nv_{th}\left(1+G_{p1}R_s\right)$$

$$\mathbf{D_{V1}} = 2I_0 + 2nv_{th}G_{p1}$$

$$\mathbf{D_{I1V1}} = 1+2\,R_S\,G_{P1}$$

$$\mathbf{D_{I2}} = -R_s\left(1+G_{p1}R_s\right)$$

$$\mathbf{D_{V2}} = -G_{p1}$$

Solving R_S, G_{p1}, *n* and I_o :

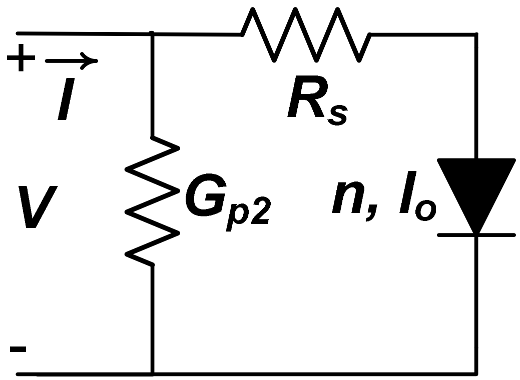
$$G_{p1} = -\mathbf{D_{V2}}$$

$$R_S = \frac{-1+\sqrt{1-4G_{P1}\mathbf{D_{I2}}}}{2G_{P1}}$$

$$n = -\frac{\mathbf{D_{I1}} + R_S\mathbf{D_{V1}}}{2v_{th}}$$

$$I_0 = \frac{\mathbf{D_{V1}} - 2nv_{th}G_{P1}}{2}$$

3.1 Function *D* vs *I* and *V* – *Particular cases*



$$\mathbf{D_{I1}} = -2R_s I_0 - 2nv_{th}$$

$$\mathbf{D_{V1}} = 2I_0\left(R_s G_{p2} + 1\right) + 2n\,v_{th} G_{p2}$$

$$\mathbf{D_{I1V1}} = 1 + 2\,R_S\,G_{P2}$$

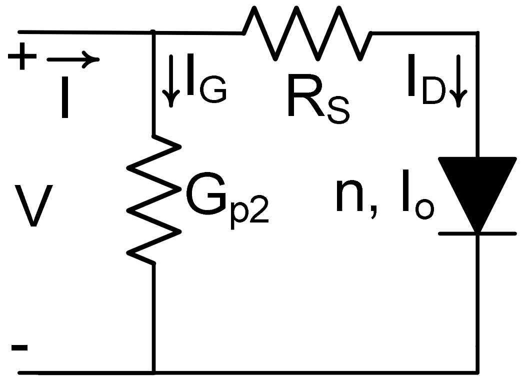
$$\mathbf{D}_{I\mathbf{2}} = -R_s \qquad \mathbf{D_{V2}} = -G_{p2}\left(1 + R_s G_{p2}\right)$$

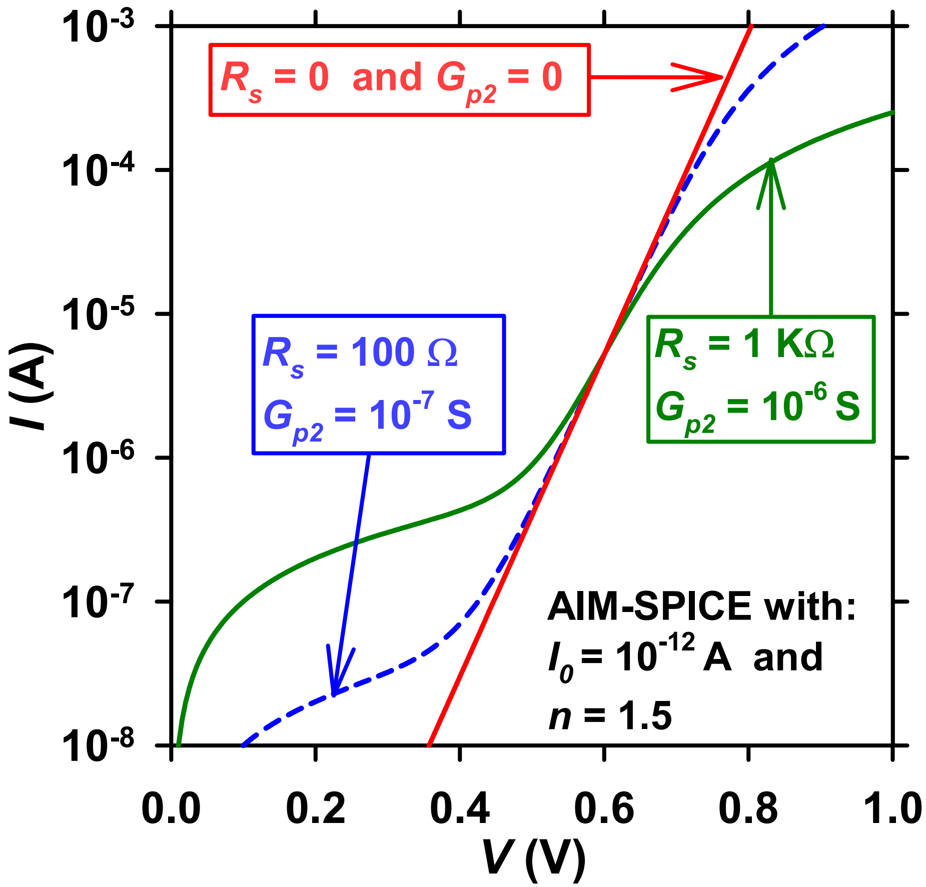
Solving R_s, G_{p2}, *n and* I_0 :

$$R_S = -\mathbf{D}_{I\mathbf{2}} \qquad G_{P2} = \frac{-1 + \sqrt{1 - 4R_s\mathbf{D_{V2}}}}{2R_s}$$

$$I_0 = \frac{\mathbf{D_{I1}} G_{P2} + \mathbf{D_{V1}}}{2} \qquad n = -\frac{2R_S I_0 + \mathbf{D_{I1}}}{2v_{th}}$$

3.2 Iterative G function method





Implicit equation

$$I = I_s \left(\exp\left[\frac{\left(1 + G_{p2} R_s\right) V - I R_s}{n\, v_{th}} \right] - 1 \right) + G_{p2} V$$

Vertical equation

$$I = \frac{n\, v_{th}}{R_S} W_0 \left\{ \frac{I_0 R_S}{n\, v_{th}} \exp\left[\frac{V + R_S I_0}{n\, v_{th}} \right] \right\} + G_{P2} V - I_0$$

Lateral equation

$$V = -\frac{n\, v_{th}}{1 + R_S G_{P2}} W_0 \left[\frac{I_0 \left(1 + R_S G_{P2}\right)}{n\, v_{th} G_{P2}} \exp\left(\frac{I + I_0 \left(1 + R_S G_{P2}\right)}{n\, v_{th} G_{P2}} \right) \right] + \frac{I + I_0}{G_{P2}}$$

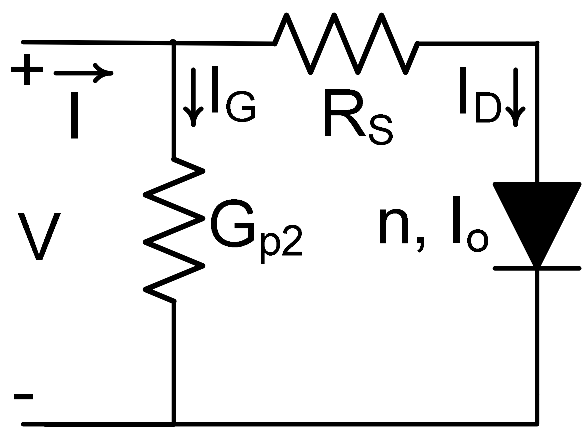
[RA00] J.C. Ranuárez, A. Ortiz-Conde, F. J. García-Sánchez, “A new method to extract diode parameters under the presence of parasitic series and shunt resistance,” Microelectronics Reliability, vol. 40, pp. 355-358 (2000). http://dx.doi.org/10.1016/S0026-2714(99)00232-2

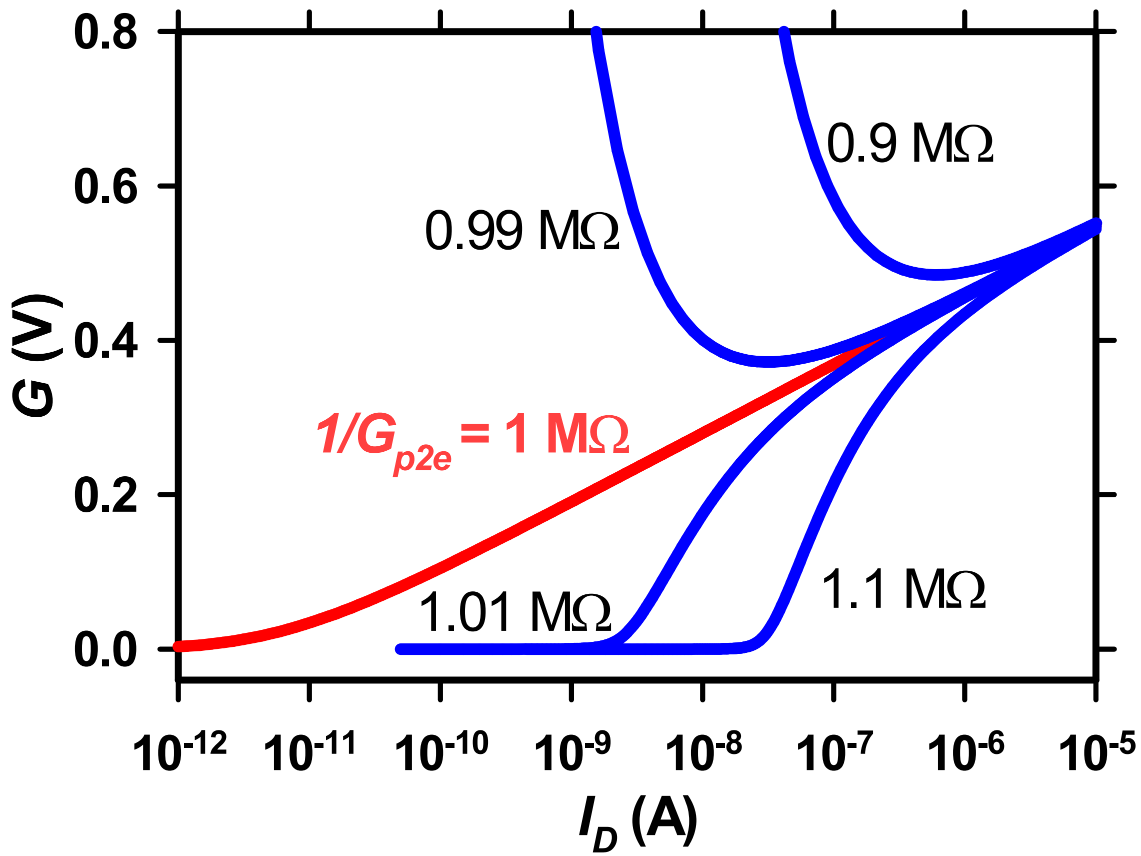
3.2 Iterative *G* function method

Plot graphs of *G* *vs* I_D for several "estimated" G_{p2e}, where

$$I_D = I - G_{p2e} V$$

$$G = \frac{D}{I_D}$$

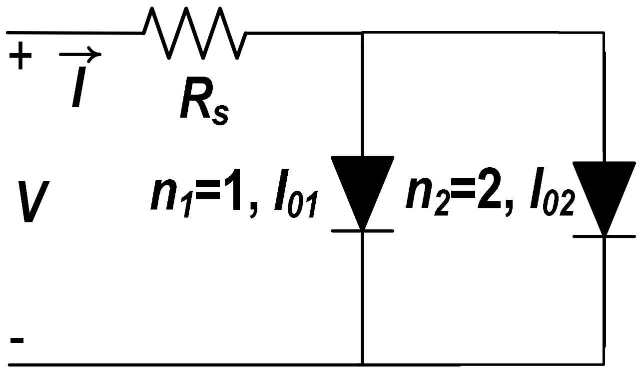




G_{p2} is the value of G_{p2e} for which the graph of *G* *vs* I_D, is a straight line.

4. Double-exponential diode models

4.1 Lateral optimization using the Quadratic Exponential Model [LU10]



$$I = I_{01}\left[\exp\left(\frac{V - I R_s}{v_{th}}\right) - 1\right] + I_{02}\left[\exp\left(\frac{V - I R_s}{2\, v_{th}}\right) - 1\right]$$

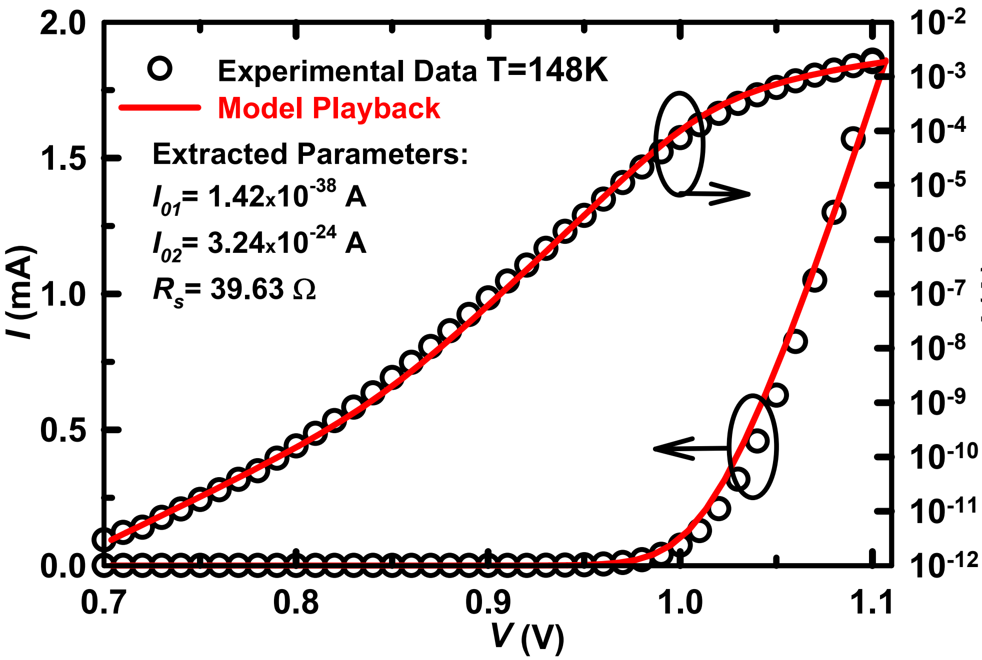
There is a solution for V and a global lateral fitting procedure is proposed to extract the parameters:

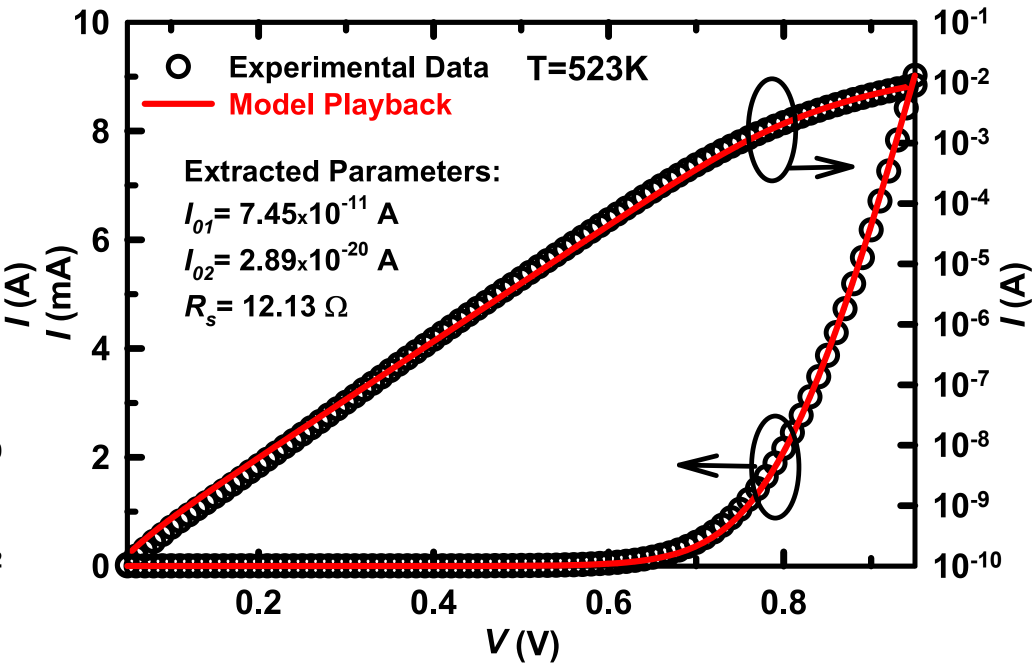
$$V = R_s\, I + 2 v_{th} \ln\left[\sqrt{\left(\frac{I_{02}}{2\, I_{01}} + 1\right)^2 + \frac{I}{I_{01}}} - \frac{I_{02}}{2\, I_{01}}\right]$$

[LU10] D. C. Lugo Muñoz, M. de Souza, M.A. Pavanello, D. Flandre. J. Muci, A. Ortiz-Conde, and F.J. García Sánchez, "Parameter Extraction in Quadratic Exponential Junction Model with Series Resistance using Global Lateral Fitting". 25th Symposium on Microelectronics Technology and Devices (SBMicro-2010). São Paulo, Brasil. Septiembre 2010. See also: Electrochemical Society Transactions, Vol. 31, pp. 369-376, 2010. http://dx.doi.org/10.1149/1.3474181

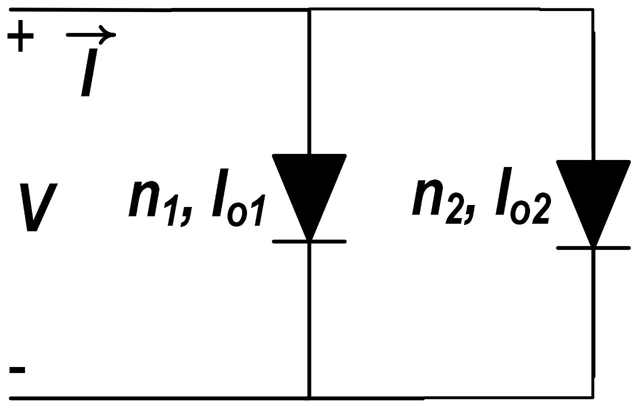
Experimental data of a silicon PIN lateral diode from the Université Catolique de Louvain

Good match between the experimental and simulated data.





4.2 Functions *A* and *B* [RA99]



Series resistance is negligible at low voltage

Traditional junction model parameter extraction methods are applied to values of the forward current considerably larger than the reverse saturation current, because they assume that the “ -1 ” term in the junction current equation may be neglected.

$$I = I_0\left(e^{V/nv_{th}} - 1\right) \approx I_0 e^{V/nv_{th}}$$

Consequently, they do not work at low forward voltages (smaller than a few thermal voltages).

[RA99] J.C. Ranuárez, F. J. García Sánchez, and A. Ortiz-Conde, “Procedure for determining diode parameters at very low forward voltage,” Solid-St. Electron., vol. 43, pp. 2129-2133, Dic. 1999. http://dx.doi.org/10.1016/S0038-1101(99)00181-1

4.2 Functions *A* and *B*

The application of either one of these operators to the experimentally measured *I-V* characteristics produces linear equations:

$$A \equiv \frac{CC}{I} = \int_0^V I\,dV \Big/ I = nv_{th} - I_0\left(\frac{V}{I}\right)$$

$$B \equiv \frac{CC}{V} = \int_0^V I\,dV \Big/ V = nv_{th}\left(\frac{I}{V}\right) - I_0$$

from whose slopes and intercepts the ideality factor and reverse saturation current parameters may be directly extracted.

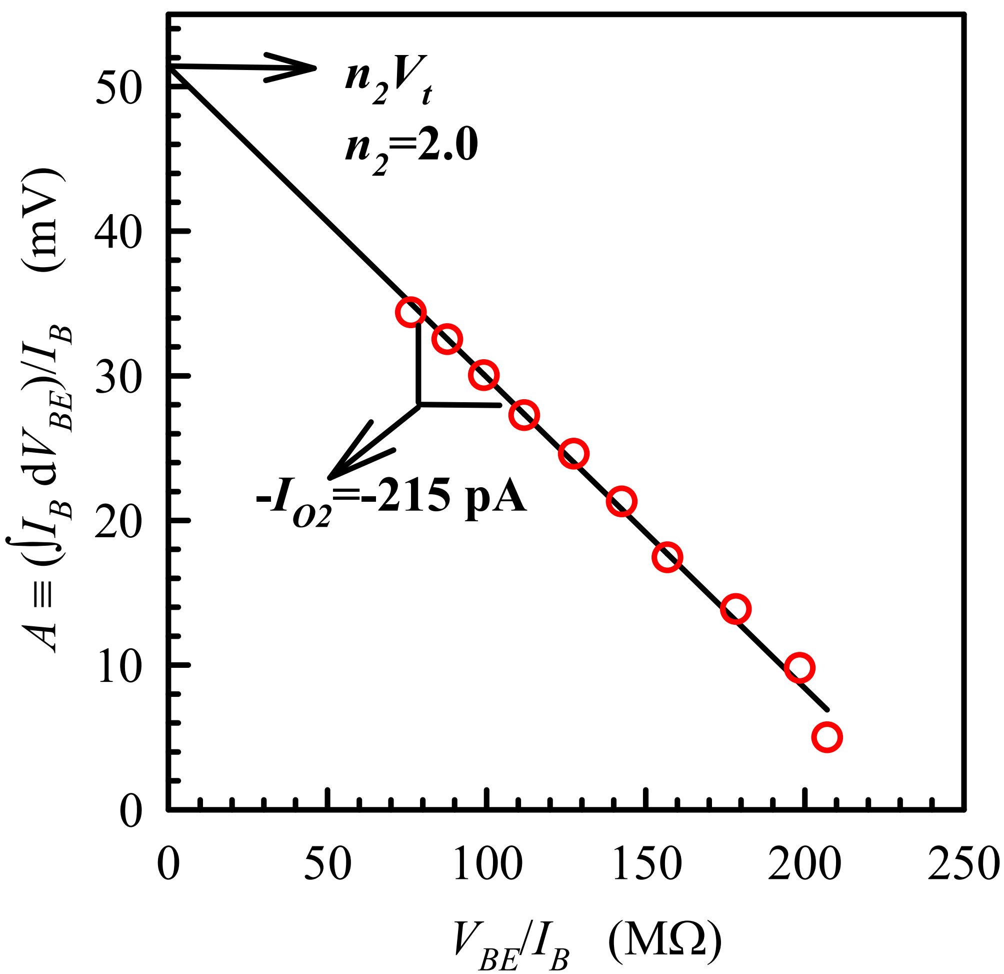
Function *A* has also been used in MOSFET parameter extraction and it is known as function *H* [OR01].

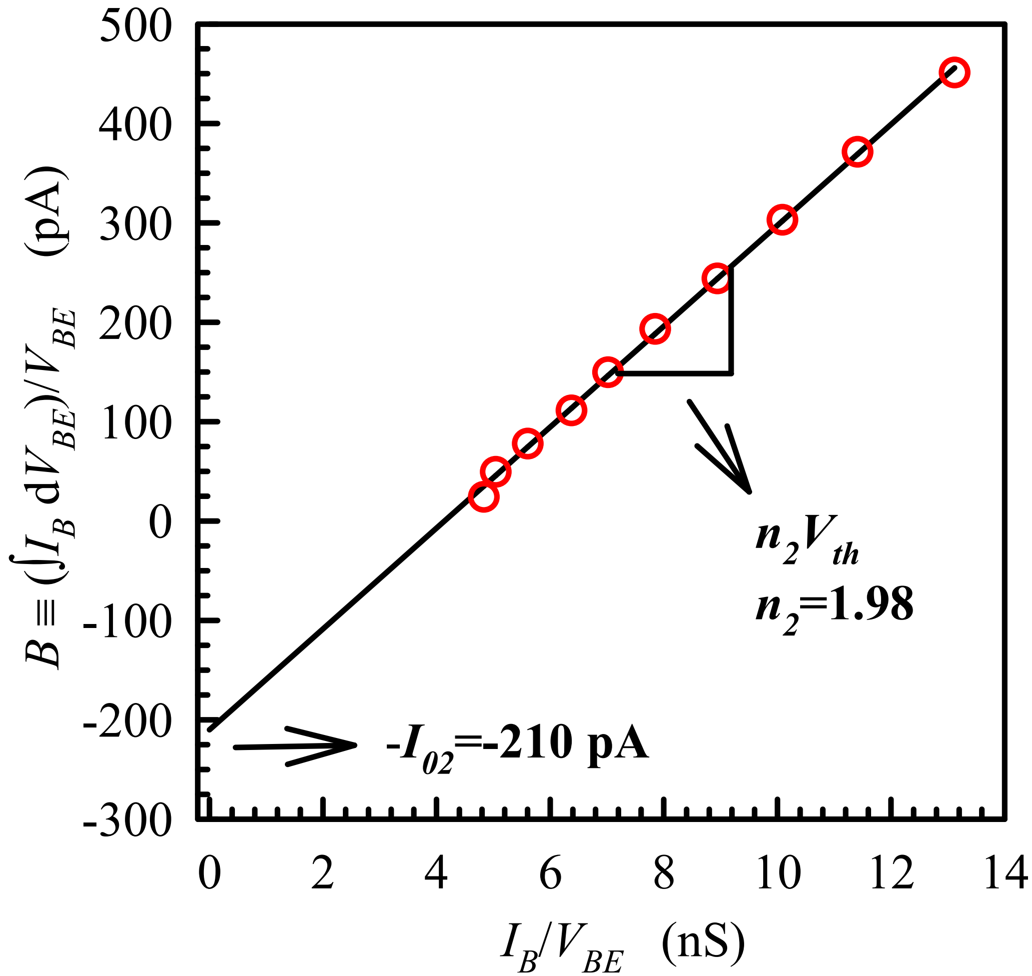
[OR01] A. Ortiz-Conde, A. Cerdeira, M. Estrada, F. J. García Sánchez, R. Quintero, "A simple procedure to extract the threshold voltage of amorphous thin film MOSFETs in the saturation region," Solid-St. Electron., vol. 45, pp. 663-667, Mayo 2001.
http://dx.doi.org/10.1016/S0038-1101(01)00123-X

4.2 Functions *A* and *B*

Example: Base current as a function of forward base-emitter voltage of a BJT (measured at T =298K with V_{BC}=0).

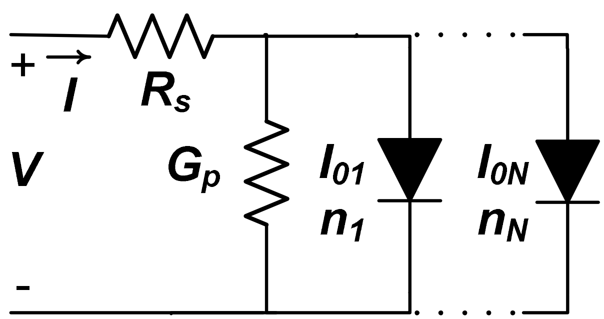
Extracted values (0<V_{BE}<100mV): I_{02} = 215pA, n_2 = 2





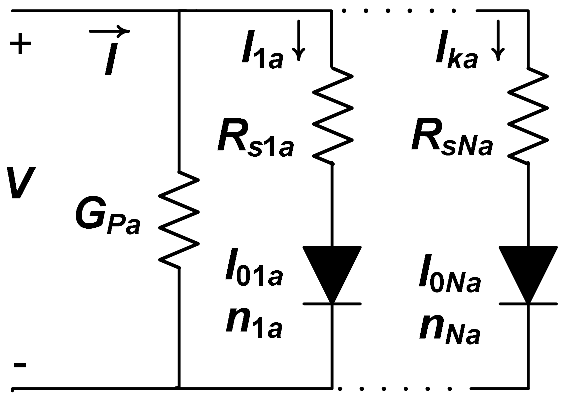
5. Multiple-exponential diode models

5.1 Alternative multi-exponential model for diode with resistances [LU11]



Conventional model requires numerical solution:

$$I = \sum_{k=1}^{N} I_{0k}\left[\exp\left(\frac{V - R_s I}{n_k v_{th}}\right) - 1\right] + G_p\left(V - R_s I\right)$$



By solving each branch separately and adding the solutions, the alternative model presents analytical solution for the current:

$$I = \sum_{k=1}^{N}\left\{\frac{n_{k\,a} v_{th}}{R_{s\,k\,a}} W_0\left[\frac{R_{s\,k\,a} I_{0k\,a}}{n_{k\,a} v_{th}} \exp\left(\frac{V + R_{s\,k\,a} I_{0k\,a}}{n_{k\,a} v_{th}}\right)\right] - I_{0ka}\right\} + G_{p\,a} V$$

where W_0 is the principal branch of the Lambert function.
Vertical optimization is used for parameter extraction.

Analytical equations are desirable for circuit simulation

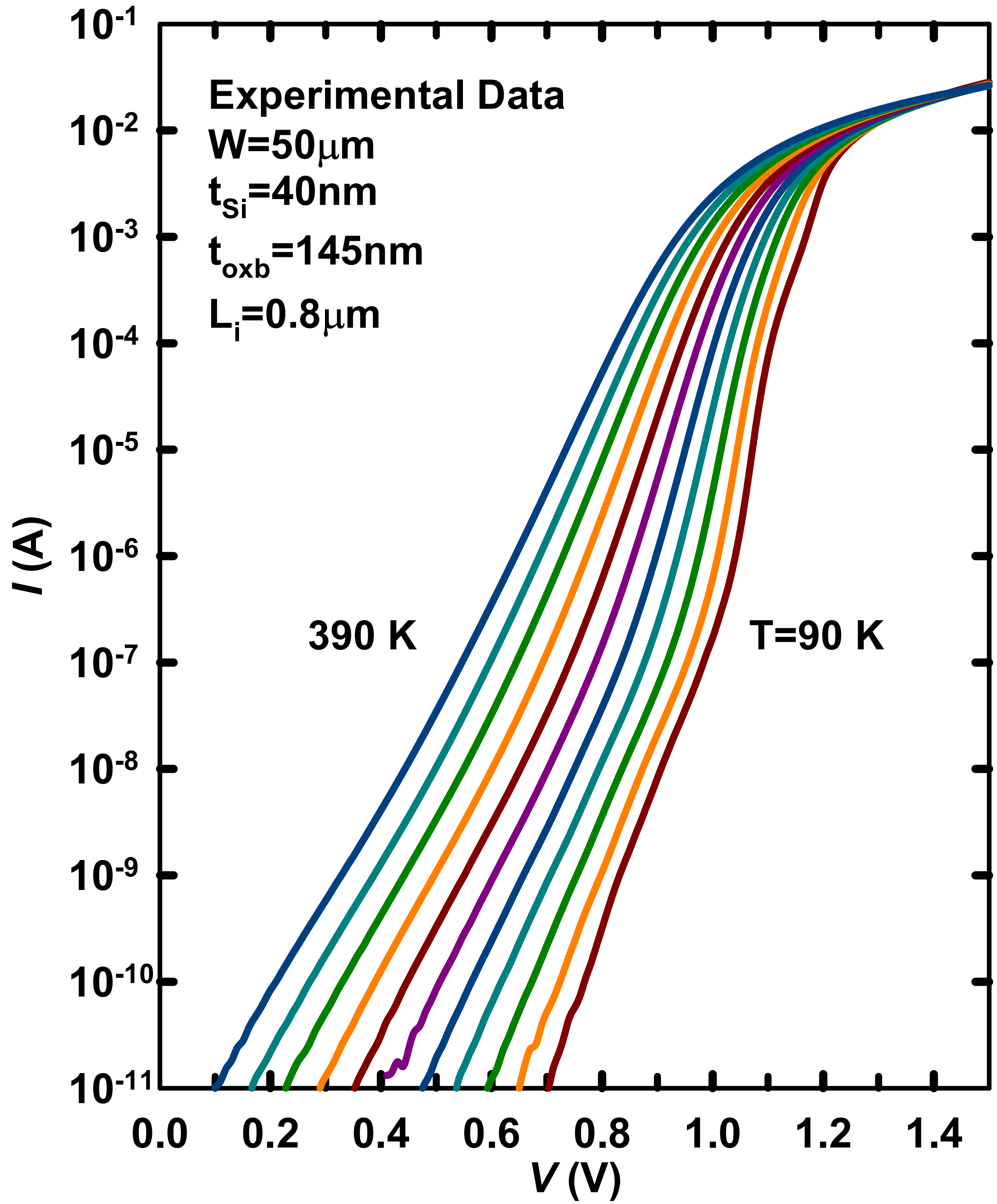
[LU11] D.C. Lugo-Muñoz, J. Muci, A. Ortiz-Conde, F. J. García Sánchez , M. de Souza, M.A. Pavanello, "An explicit multi-exponential model for semiconductor junctions with series and shunt resistances", Microelectronics Reliability, Vol. 51 pp. 2044–2048, Diciembre de 2011. http://dx.doi.org/10.1016/j.microrel.2011.06.030

5.1 Alternative multi-exponential model for diode with resistances.

Measured I-V characteristics of a lateral SOI PIN diode for a wide range of operating temperatures [LU11b,12] .

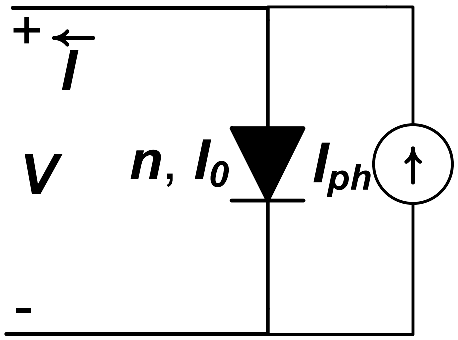
[LU11b] D.C. Lugo-Muñoz, J. Muci, A. Ortiz-Conde, F.J. García Sánchez, M. de Souza, D. Flandre, M.A. Pavanello, "Characterization of thin-film SOI PIN diodes from cryogenic to above room using an explicit I-V multi-Branch model", ECS Transactions, Vol. 39 (1), pp. 171-178, 2011. http://dx.doi.org/10.1149/1.3615191

[LU12] D.C. Lugo-Muñoz, J. Muci, A. Ortiz-Conde, F.J. García-Sánchez, M. de Souza, D. Flandre y M. A. Pavanello, "Modeling of Thin-Film Lateral SOI PIN Diodes with an Alternative Multi-Branch Explicit Current Model", Journal of Integrated Circuits and Systems (JICS), vol. 7, no. 1, pp. 92-99, 2012.



6. Single-exponential solar cell model

6.1. without resistances



Only 3 parameters:
n, I_o, I_{ph}

$$I = I_{ph} - I_0\left[\exp\left(\frac{V}{n\,v_{th}}\right)-1\right]$$

$$I_{SC} \equiv I\big|_{V=0} = I_{ph}$$

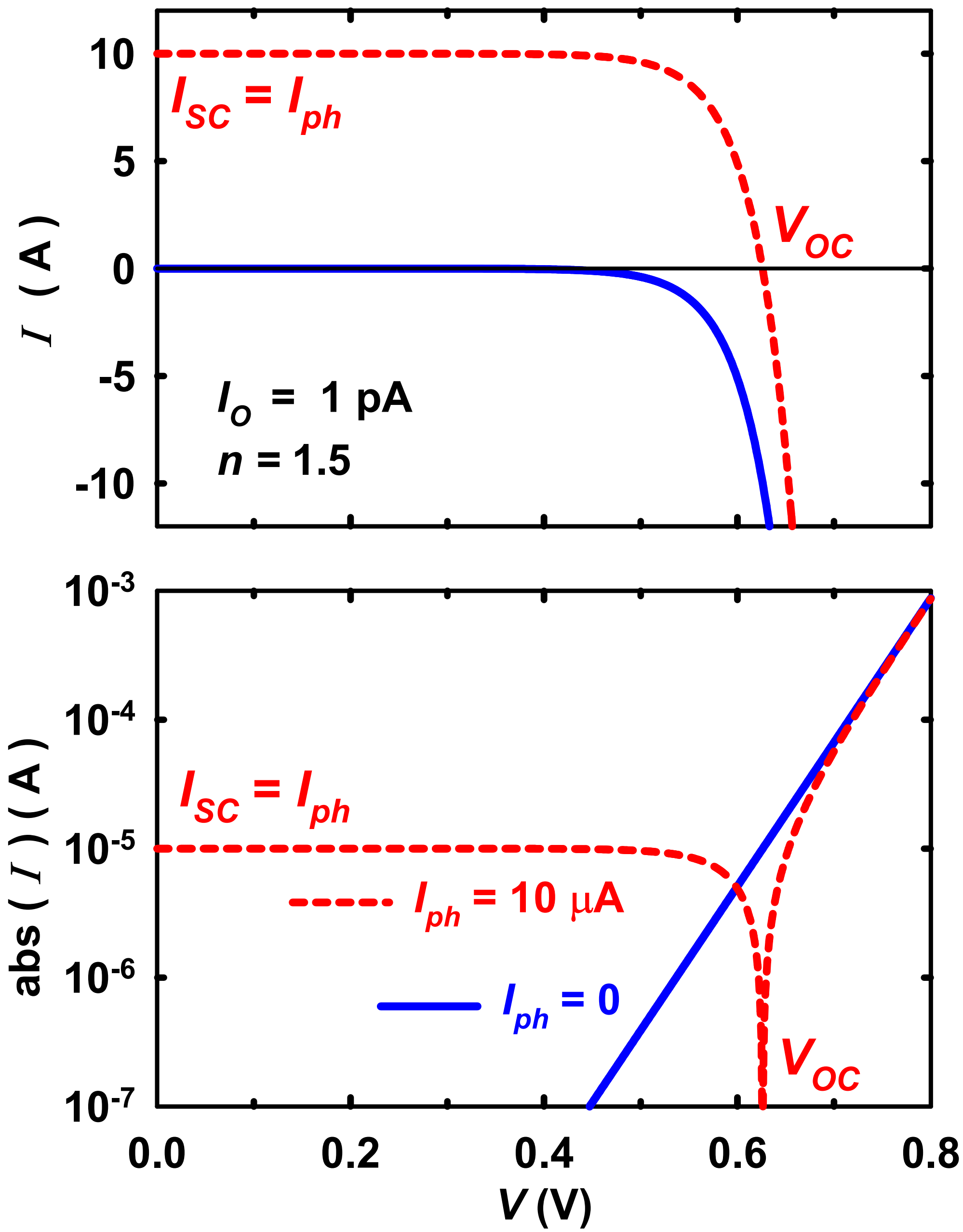
$$V = n\,v_{th}\,ln\left(1+\frac{I_{ph}-I}{I_0}\right)$$

$$V_{OC} \equiv V\big|_{I=0} = n\,v_{th}\,ln\left(1+\frac{I_{ph}}{I_0}\right)$$

$$\approx n\,v_{th}\,ln\left(\frac{I_{ph}}{I_0}\right)$$

$$\left.\frac{dI}{dV}\right|_{V=0\,,\,I=I_{SC}} = -\frac{I_0}{n\,v_{th}}$$

$$\left.\frac{dI}{dV}\right|_{V=V_{OC}\,,\,I=0} = -\frac{I_{ph}+I_0}{n\,v_{th}}$$



6.1. Single-exponential model without resistance

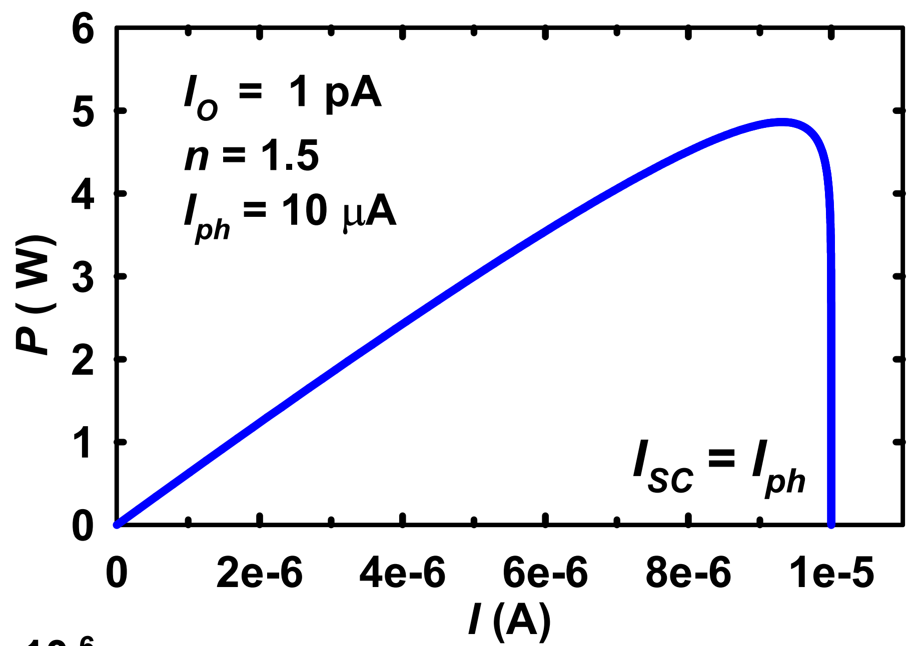
The power is:

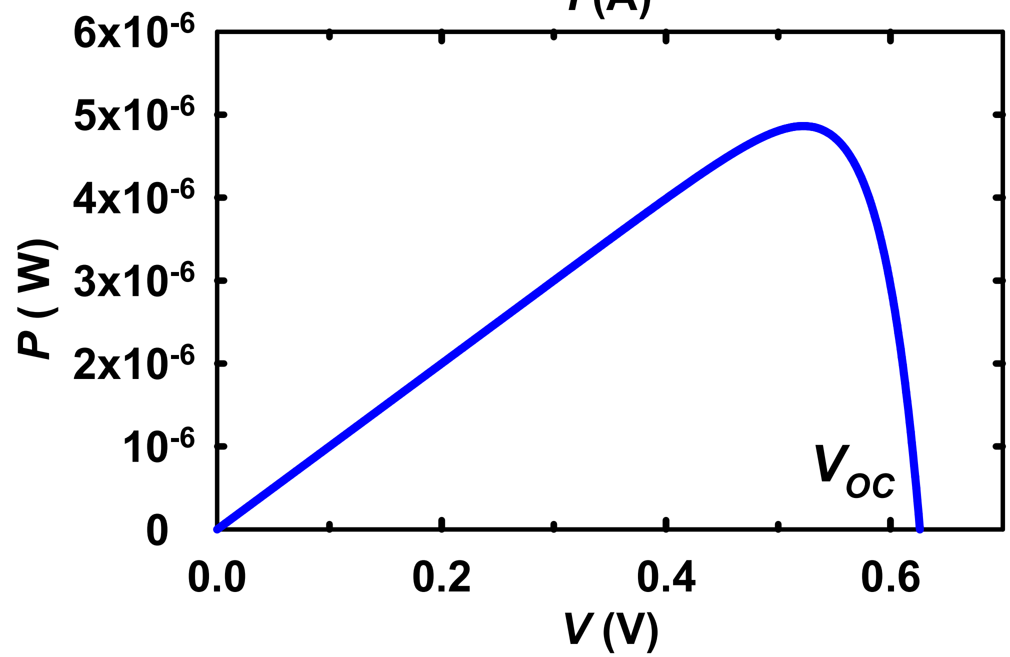
$$P = V\,I = V\left\{ I_{ph} - I_o\left[\exp\left(\frac{V}{n\,v_{th}}\right) - 1\right]\right\}$$

The value of the voltage for maximum power is obtained by deriving the previous equation and equating it to zero:

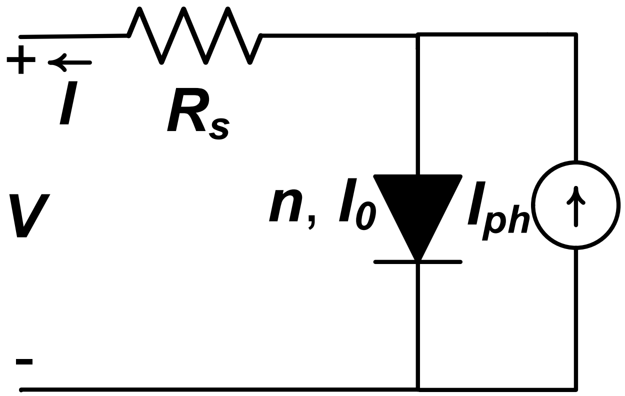
$$\boxed{V_{\max} = n\,v_{th}\left[W\left(\frac{e\left(I_0 + I_{ph}\right)}{I_0}\right) - 1\right] \approx n\,v_{th}\left[W\left(\frac{2.718 I_{ph}}{I_0}\right) - 1\right]}$$

$$\boxed{\begin{aligned} I_{\max} &= I_{ph} - I_0\left\{\exp\left[W\left(\frac{e\left(I_0 + I_{ph}\right)}{I_0}\right) - 1\right] - 1\right\} \\ &\approx I_{ph} - I_0\left\{\exp\left[W\left(\frac{eI_{ph}}{I_0}\right) - 1\right] - 1\right\}\end{aligned}}$$





6.2. Single-exponential model with series resistance



Implicit equation

$$I = I_{ph} - I_o\left[\exp\left(\frac{V + R_S I}{n v_{th}}\right) - 1\right]$$

Lateral equation

$$V = -R_S I + n v_{th} \ln\left(1 + \frac{I_{ph} - I}{I_0}\right)$$

4 parameters: *n, I_o, I_{ph}, R_S*

$$V_{OC} \equiv V\big|_{I=0} = n v_{th} \ln\left(1 + \frac{I_{ph}}{I_0}\right)$$

Vertical equation [JA04]:

$$I = (I_{ph} + I_0) - \frac{n v_{th}}{R_S} W\left[\exp\left[\frac{V + R_S(I_{ph} + I_0)}{n v_{th}} + \ln\left(\frac{I_0 R_S}{n v_{th}}\right)\right]\right]$$

$$= (I_{ph} + I_0) - \frac{n v_{th}}{R_S}\, \omega\left[\frac{V + R_S(I_{ph} + I_0)}{n v_{th}} + \ln\left(\frac{I_0 R_S}{n v_{th}}\right)\right]$$

$$I_{SC} = I\big|_{V=0} = (I_{ph} + I_0) - \frac{n v_{th}}{R_S} W\left[\exp\left[\frac{R_S(I_{ph} + I_0)}{n v_{th}} + \ln\left(\frac{I_0 R_S}{n v_{th}}\right)\right]\right]$$

$$= (I_{ph} + I_0) - \frac{n v_{th}}{R_S}\, \omega\left[\frac{R_S(I_{ph} + I_0)}{n v_{th}} + \ln\left(\frac{I_0 R_S}{n v_{th}}\right)\right]$$

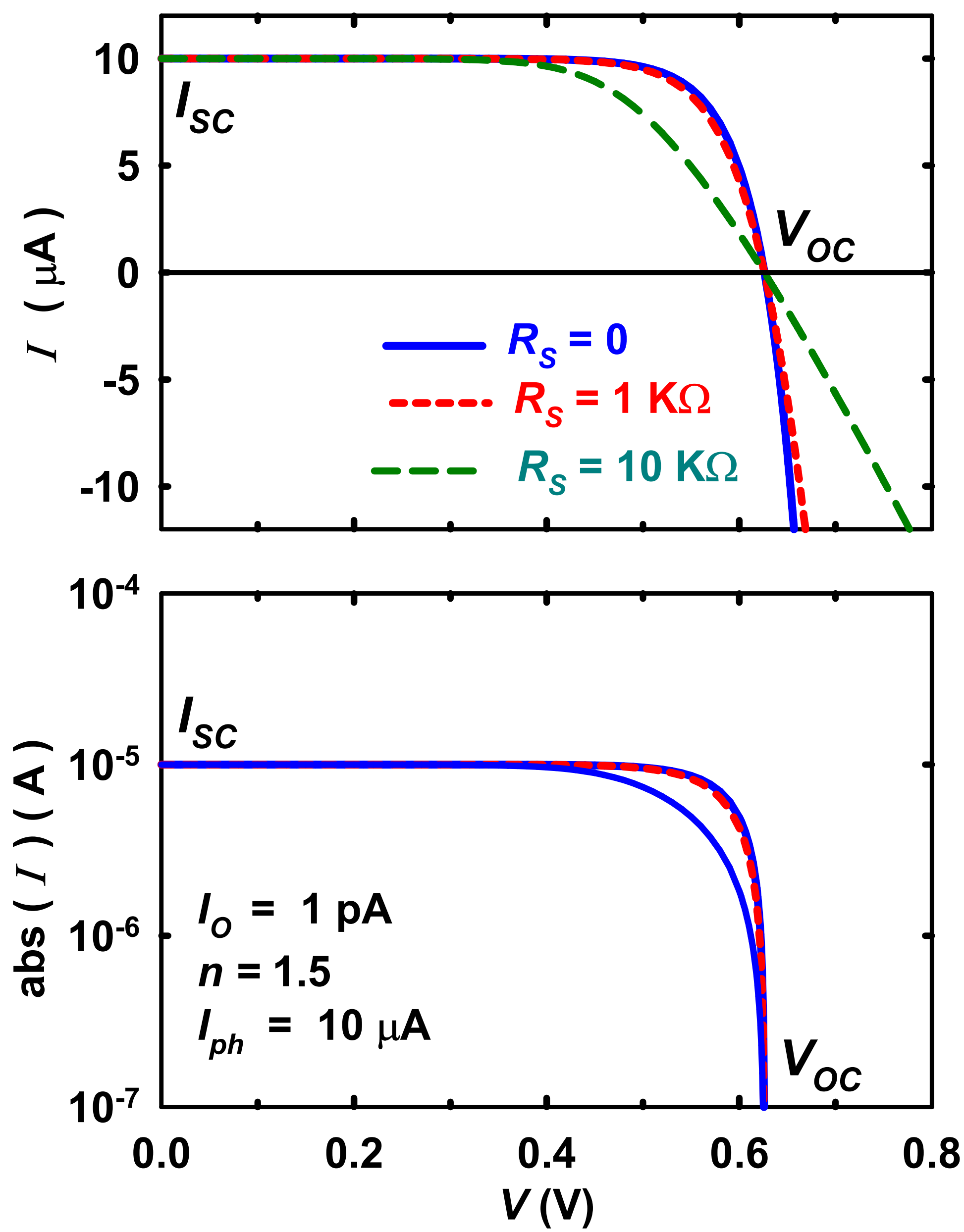
[JA04] A. Jain, A. Kapoor, "Exact analytical solutions of the parameters of real solar cells using Lambert W-function", Solar Energy Materials and Solar Cells 81 (2), pp. 269-277, 2004. http://dx.doi.org/10.1016/j.solmat.2003.11.018

6.2. Single-exponential model with series resistance

V_{oc} does not depend on R_s
R_s is important close to V_{oc}
I_{sc} is approximately equal to I_{ph}

$$\left.\frac{dV}{dI}\right|_{V=0\,,\,I=I_{SC}} = -R_S - \frac{n\,v_{th}}{I_{ph}+I_0-I_{SC}}$$

$$\left.\frac{dV}{dI}\right|_{V=V_{OC}\,,\,I=0} = -R_S - \frac{n\,v_{th}}{I_{ph}+I_0}$$



Lateral optimization method can be used:

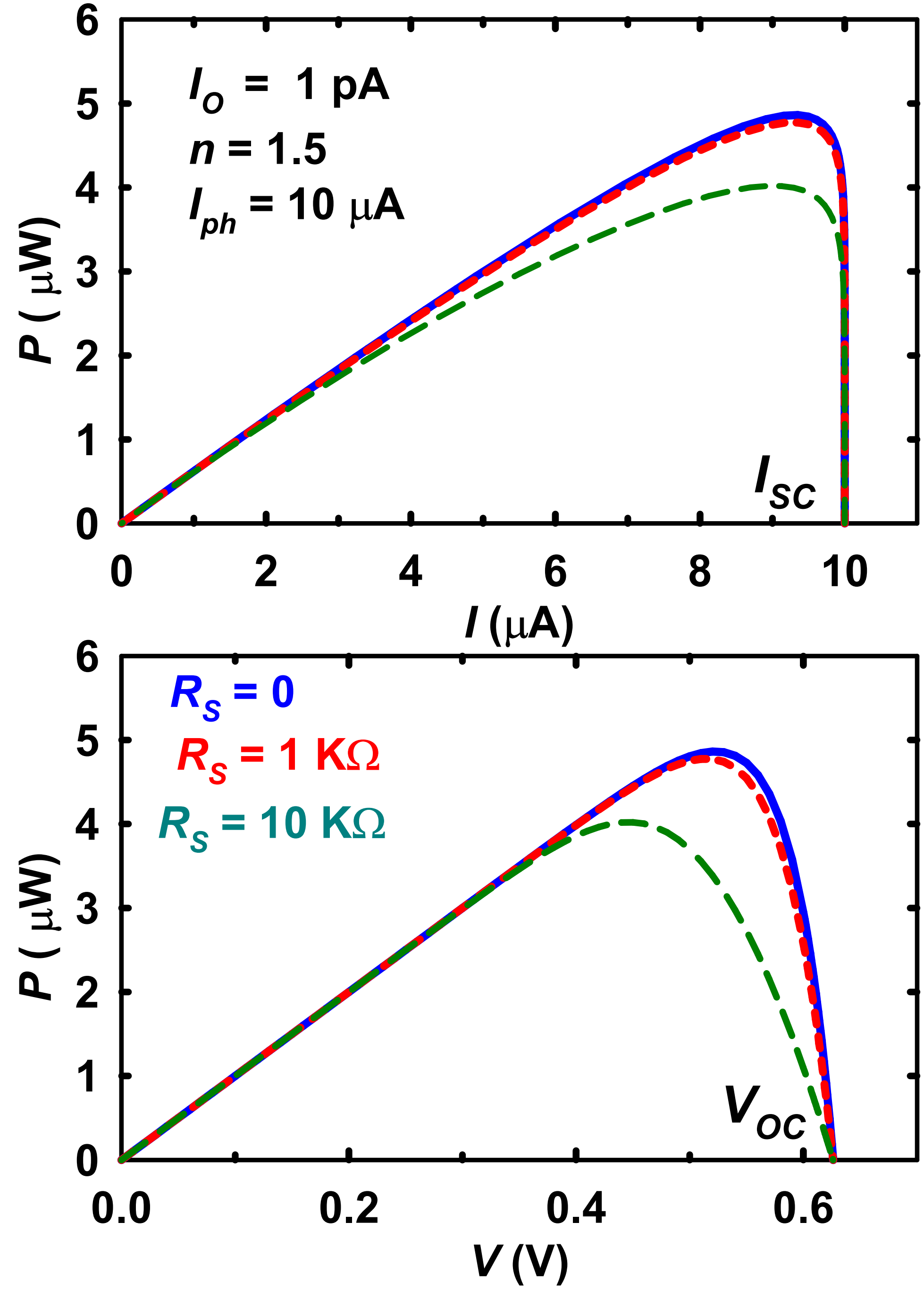
$$V = -R_S\,I + n\,v_{th}\,\ln\left(1+\frac{I_{ph}-I}{I_0}\right)$$

6.2. Single-exponential model with series resistance

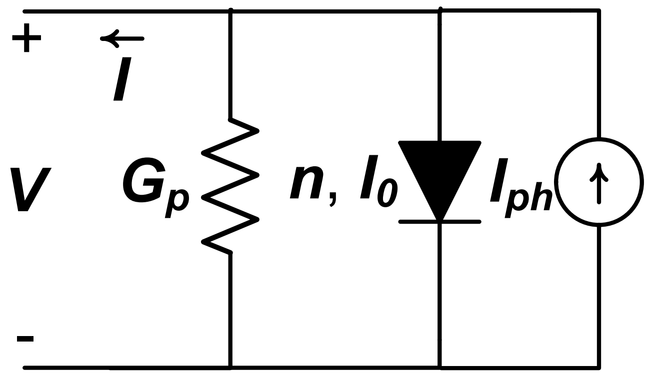
This case does not have analytical solution for the maximum power point.

The power decreases as R_s increases.

The location of the maximum power depends of R_s.



6.3. Single-exponential model with parallel resistance



Lateral equation

$$V = \frac{(I_{ph}+I_0-I)}{G_p} - nv_{th}\, W\left[exp\left[\frac{(I_{ph}+I_0-I)}{G_p n v_{th}} + ln\left(\frac{I_0}{G_p n v_{th}}\right)\right]\right]$$

$$= \frac{(I_{ph}+I_0-I)}{G_p} - nv_{th}\, \omega\left[\frac{(I_{ph}+I_0-I)}{G_p n v_{th}} + ln\left(\frac{I_0}{G_p n v_{th}}\right)\right]$$

4 parameters: *n, I_o, I_{ph}, G_p*

$$V_{OC} = V|_{I=0} = V$$

$$= \frac{(I_{ph}+I_0)}{G_p} - nv_{th}\, W\left[exp\left[\frac{(I_{ph}+I_0)}{G_p n v_{th}} + ln\left(\frac{I_0}{G_p n v_{th}}\right)\right]\right]$$

$$= \frac{(I_{ph}+I_0)}{G_p} - nv_{th}\, \omega\left[\frac{(I_{ph}+I_0)}{G_p n v_{th}} + ln\left(\frac{I_0}{G_p n v_{th}}\right)\right]$$

Vertical equation

$$\boxed{I = I_{ph} - G_p V - I_o\left[\exp\left(\frac{V}{n v_{th}}\right) - 1\right]}$$

$$I_{SC} \equiv I\big|_{V=0} = I_{ph}$$

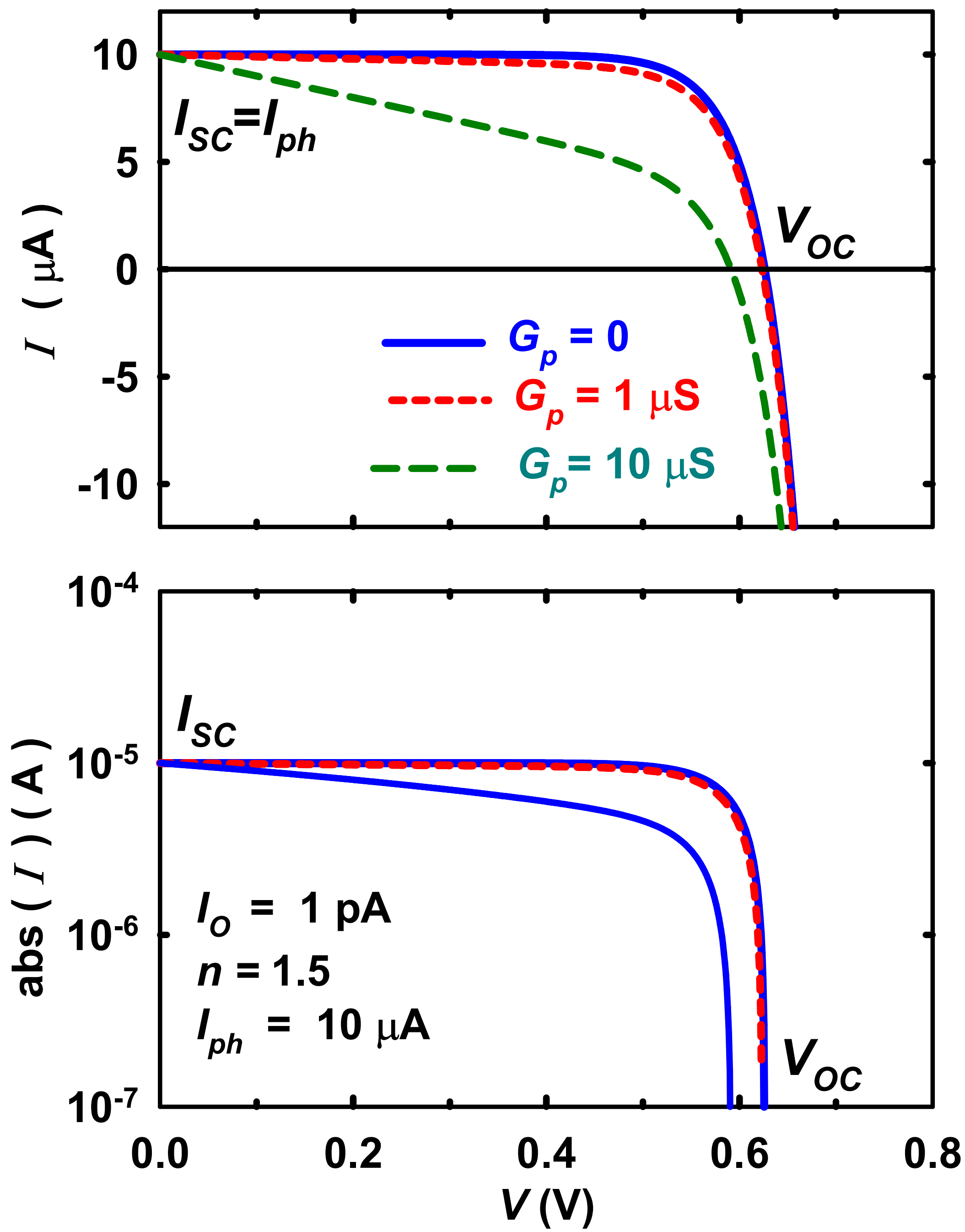
$$\left.\frac{dI}{dV}\right|_{V=0\,,\,I=I_{SC}} = -G_p - \frac{I_0}{n v_{th}}$$

$$\left.\frac{dI}{dV}\right|_{V=V_{OC}\,,\,I=0} = -G_p - \frac{I_0}{n v_{th}}\exp\left(\frac{V_{OC}}{n v_{th}}\right)$$

6.3 Single-exponential model with parallel resistance

$I_{SC} = I_{ph}$ is a constant but G_p is important close to I_{SC}

This case does not have analytical solution for the maximum power point.

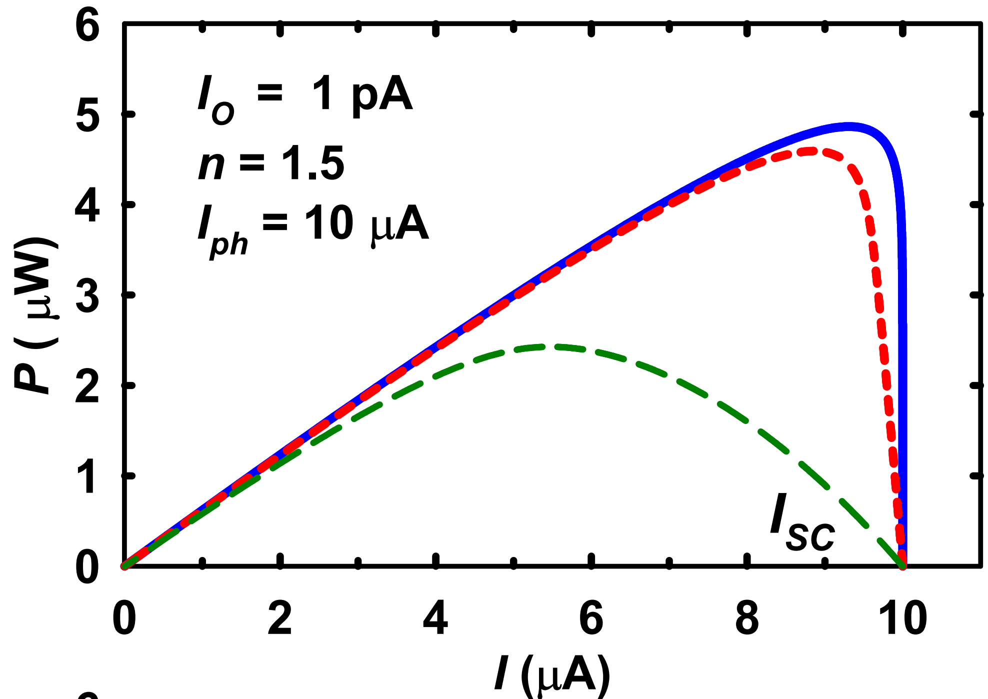


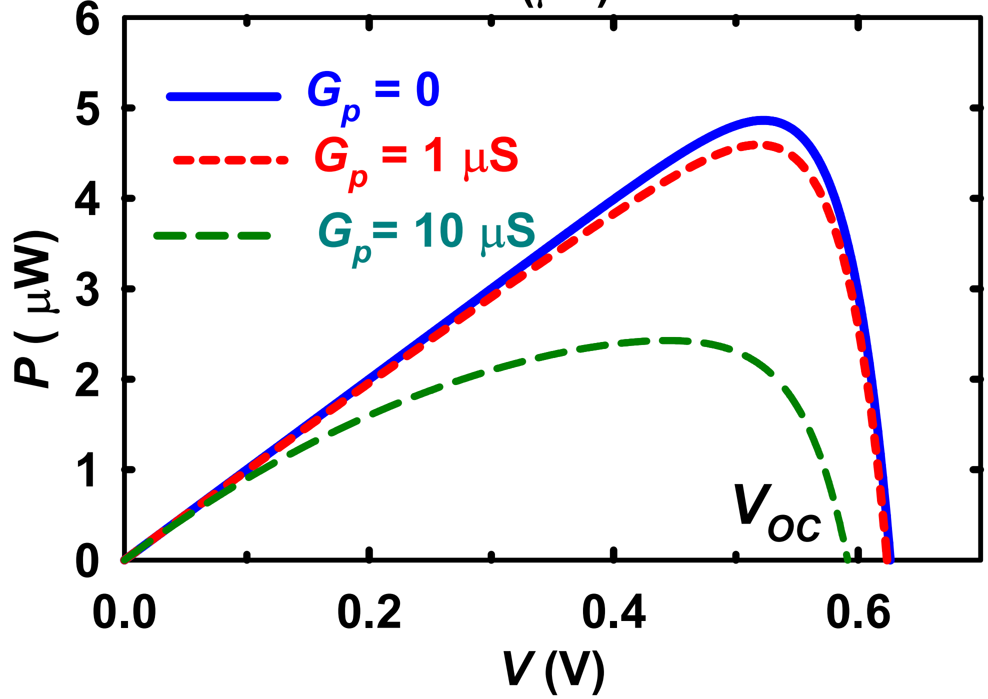
6.3 Single-exponential model with parallel resistance

This case does not have analytical solution for the maximum power point.

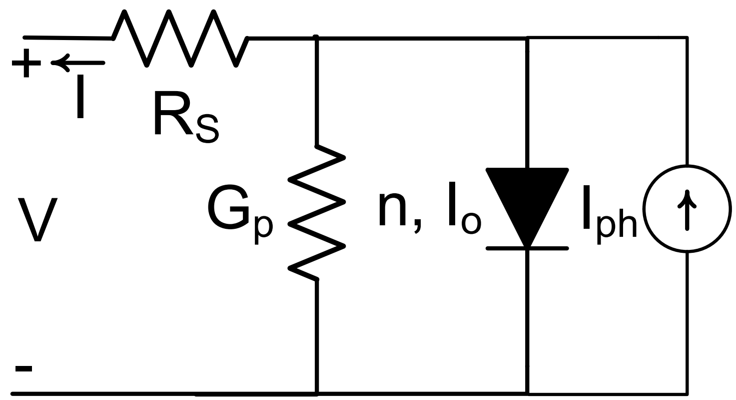
The power decreases as G_p increases.

The location of the maximum power depends of G_p.





6.4 Model with 5 parameters: *n, I_o, I_{ph}, R_s, G_p*.



This case does not have analytical solution for the maximum power point.

Implicit equation

$$I=-I_o\left[\exp\left(\frac{V+I\,R_s}{n\,V_{th}}\right)-1\right]-\left(V+I\,R_s\right)G_p+I_{ph}$$

Vertical Equation [JA04]:

$$I=\frac{(I_{ph}+I_0)-G_pV}{1+R_SG_p}-\frac{nv_{th}}{R_S}W\left[exp\left[\frac{V+R_S(I_{ph}+I_0)}{nv_{th}(1+R_SG_p)}+ln\left(\frac{I_0R_S}{nv_{th}(1+R_SG_p)}\right)\right]\right]$$

$$=\frac{(I_{ph}+I_0)-G_pV}{1+R_SG_p}-\frac{nv_{th}}{R_S}\,\omega\left[\frac{V+R_S(I_{ph}+I_0)}{nv_{th}(1+R_SG_p)}+ln\left(\frac{I_0R_S}{nv_{th}(1+R_SG_p)}\right)\right]$$

Lateral equation

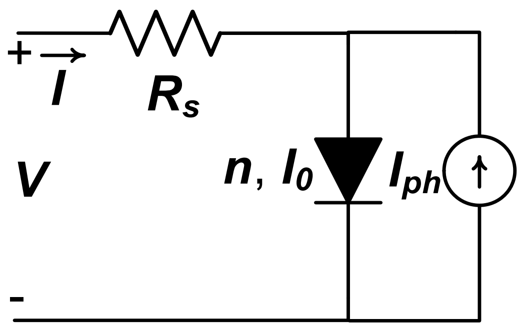
$$V=I\left(R_S+\frac{1}{G_p}\right)+\frac{(I_{ph}+I_0)}{G_p}-nv_{th}W\left[exp\left[\frac{(I_{ph}+I_0-I)}{nv_{th}G_p}+ln\left(\frac{I_0}{nv_{th}G_p}\right)\right]\right]$$

$$=I\left(R_S+\frac{1}{G_p}\right)+\frac{(I_{ph}+I_0)}{G_p}-nv_{th}\,\omega\left[\frac{(I_{ph}+I_0-I)}{nv_{th}G_p}+ln\left(\frac{I_0}{nv_{th}G_p}\right)\right]$$

[JA04] A. Jain, A. Kapoor, "Exact analytical solutions of the parameters of real solar cells using Lambert W-function", Solar Energy Materials and Solar Cells 81 (2), pp. 269-277, 2004. http://dx.doi.org/10.1016/j.solmat.2003.11.018

7. Parameter extraction methods in solar cells

7.1 First integration method to extract R_s for solar cells [AR82]



$$V = +I\,R_S + nV_T\,\ln\left(\frac{I+I_{ph}}{I_0}+1\right)$$

For $I >> I_o$ and $I_{ph} >> I_o$:

$$\int_0^I V\,dI \approx \frac{R_S}{2}I^2 - n\,v_{th}\,I + n\,v_{th}\left(I+I_{ph}\right)\ln\left(\frac{I+I_{ph}}{I_o}\right) - n\,v_{th}\,I_{ph}\,\ln\left(\frac{I_{ph}}{I_o}\right)$$

Then, changing variables R_s is evaluated:

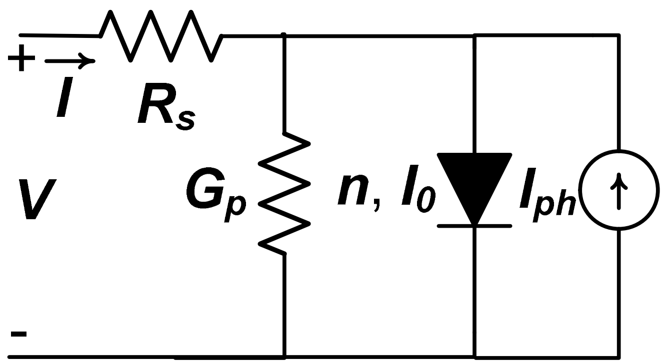
$$R_S \approx 2\frac{\int_0^{I_{sc}} V\,dI}{I_{SC}^2} + 2\frac{n\,v_{th}}{I_{SC}} - 2\frac{V_{oc}}{I_{SC}}$$

where $I_{SC} \approx -I_{ph}$

$$V_{oc} = n\,v_{th}\,ln\left(1+\frac{I_{ph}}{I_0}\right) \approx n\,v_{th}\,\ln\left(-\frac{I_{SC}}{I_0}\right)$$

[AR82] G. L. Araujo and E. Sanchez, "New method for experimental determination of the series resistance of a solar cell", IEEE Transactions on Electron Devices ED-29 (10) , pp. 1511-1513, 1982. http://dx.doi.org/10.1109/T-ED.1982.20906

7.2 Vertical Optimization [PO12]

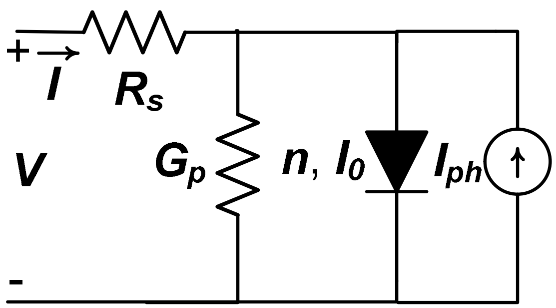


$$I = \frac{nv_{th}}{R_S} W\left\{ \frac{I_0 R_S}{nv_{th}\left(1+R_S\ G_P\right)} \exp\left[\frac{V+R_S\left(I_0+I_{ph}\right)}{nv_{th}\left(1+R_S\ G_P\right)} \right] \right\} + \frac{VG_P-\left(I_0+I_{ph}\right)}{1+R_S\ G_P}$$

Del Pozo et al fitted the previous equation to the experimental data using the non-linear curve fitting routine lsqcurvefit implemented in MATLAB.

[PO12] Del Pozo, G., Romero, B., Arredondo, B., "Extraction of circuital parameters of organic solar cells using the exact solution based on Lambert W-function", Proceedings of SPIE - The International Society for Optical Engineering 8435 , art. no. 84351Z, 2012. http://dx.doi.org/10.1117/12.922461

7.3 Integration of the Co-content Function [OR06]



Integrating

$$\boxed{CC(I,V) \equiv \int_0^V \left(I - I_{sc}\right)\, dV}$$

where:

$$I = \frac{n v_{th}}{R_S} W \left\{ \frac{I_0 R_S}{n v_{th}\left(1 + R_S\, G_P\right)} \exp\left[\frac{V + R_S\left(I_0 + I_{ph}\right)}{n v_{th}\left(1 + R_S\, G_P\right)} \right] \right\} + \frac{V G_P - \left(I_0 + I_{ph}\right)}{1 + R_S\, G_P}$$

$$V = -\, n v_{th} W \left[\frac{I_0}{n v_{th} G_P} \exp\left(\frac{I + I_0 + I_{ph}}{n v_{th} G_P} \right) \right] + I\left(R_S + \frac{1}{G_P} \right) + \frac{I_0 + I_{ph}}{G_P}$$

After performing the integration and doing some algebraic manipulations, we obtain function CC in the form of a convenient purely algebraic expression:

$$CC(I,V) = C_{V1}\, V + C_{I1}\left(I - I_{sc}\right) + C_{I1V1}\, V\left(I - I_{sc}\right) + C_{V2}\, V^2 + C_{I2}\left(I - I_{sc}\right)^2$$

[OR06] A. Ortiz-Conde, F. J. García Sánchez , and J. Muci, "New method to extract the model parameters of solar cells from the explicit analytic solutions of their illuminated I-V characteristics", Solar Energy Materials and Solar Cells, Vol. 90, pp. 352–361, Feb. 2006. http://dx.doi.org/10.1016/j.solmat.2005.04.023

7.3 Integration of the Co-content Function

where the coefficients are:

$$C_{I1} = R_S\left(I_0 + I_{ph} + I_{sc}\right) + nv_{th}(1+G_P R_S) + I_{sc}\ R_S^2\ G_P$$

$$C_{V1} = -\left(I_0 + I_{ph} + I_{sc}\right) - nv_{th} G_P - I_{sc}\ R_S\ G_P$$

$$C_{I2} = \frac{R_S\left(1+G_P R_S\right)}{2} \qquad C_{V2} = \frac{G_P}{2}$$

Unfortunately, the fifth coefficient is dependent on the others:

$$\boxed{C_{I1V1} = \frac{1 - \sqrt{1+16\,C_{I2}\,C_{V2}}}{2}}$$

7.3 Integration of the Co-content Function

There are actually four independent coefficients and therefore only four unknowns may be extracted uniquely. The general solution of G_p, R_S, n and I_{PH}, in terms of C_{I1}, C_{V1}, C_{I2}, C_{V2}, is:

$$G_P = 2\ C_{V2}$$

$$R_S = \frac{\sqrt{1+16\,C_{V2}\,C_{I2}}-1}{4\,C_{V2}}$$

$$n = \frac{C_{V1}\left(\sqrt{1+16\,C_{V2}\,C_{I2}}-1\right)+4\,C_{I1}\,C_{V2}}{4\,V_{th}\,C_{V2}}$$

Assuming:

$I_0 << I_{ph}$

$$I_{ph} = -\frac{\left(1+\sqrt{1+16\,C_{V2}\,C_{I2}}\right)\left(C_{V1}+I_{sc}\right)}{2} - 2\,C_{I1}\,C_{V2}$$

Finally, the value of I_o is obtained by using implicit equation:

$$I_o = \frac{I-(V-I\,R_s)\,G_p+I_{ph}}{\exp\left(\dfrac{V-I\,R_s}{n\,V_{th}}\right)-1}$$

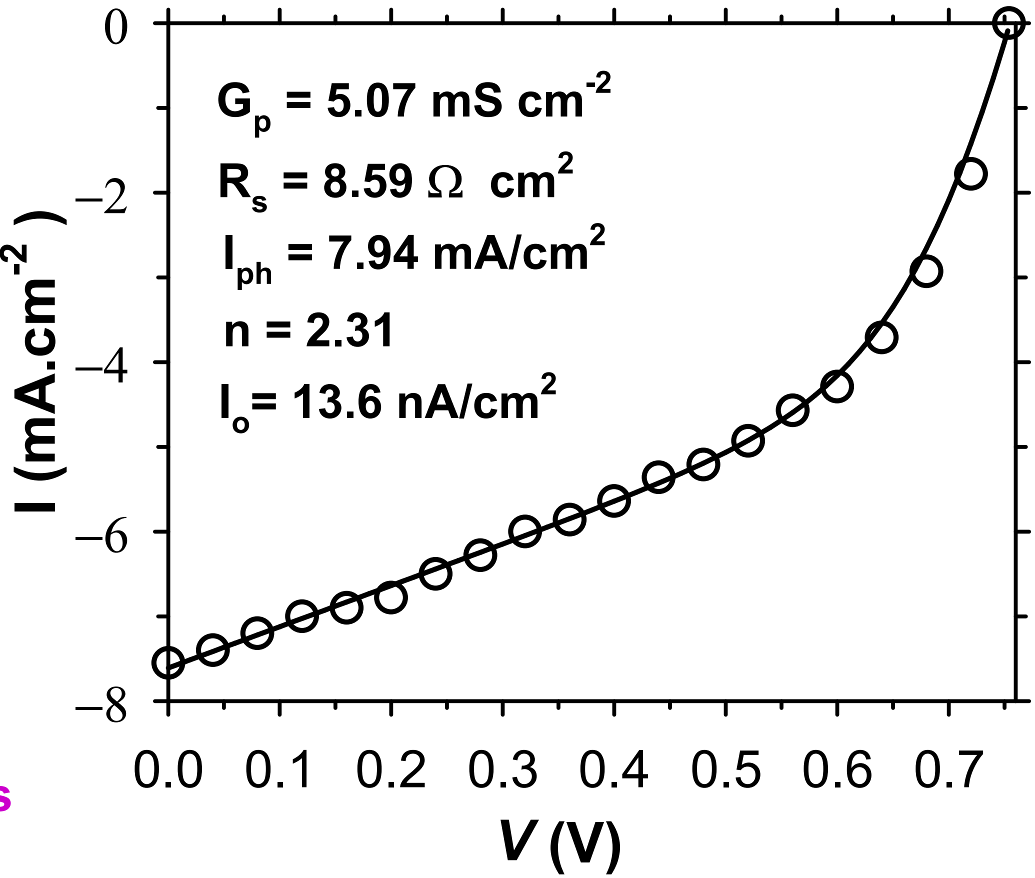
7.3 Integration of the Co-content Function

Bivariate fitting of function *D* in the form of this algebraic expression:

$$CC(I,V) = C_{V1} V + C_{I1}\left(I - I_{sc}\right) + C_{I1V1} V \left(I - I_{sc}\right) + C_{V2} V^2 + C_{I2}\left(I - I_{sc}\right)^2$$

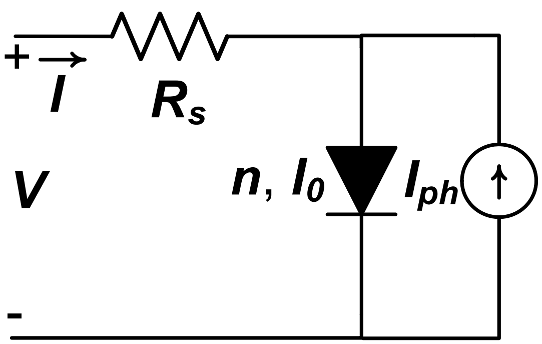
to the experimental data produces the corresponding coefficients, from which the four diode model parameters can be calculated.

Experimental data from a plastic solar cell [JE04] was used to test the procedure.



[JE04] Jeranko, T., Tributsch, H., Sariciftci, N.S., Hummelen, J.C., "Patterns of efficiency and degradation of composite polymer solar cells", Solar Energy Materials and Solar Cells 83 (2-3) , pp. 247-262, 2004.
http://dx.doi.org/10.1016/j.solmat.2004.02.028

7.3 Integration of the Co-content Function – *Particular cases*



$$\mathrm{C}_{\mathrm{I2}} = \frac{R_S}{2}$$

$$\mathrm{C}_{\mathrm{I1}} = R_S\left(I_0 + I_{ph} + I_{sc}\right) + nv_{th}$$

$$\mathrm{C}_{\mathrm{V1}} = -\left(I_0 + I_{ph} + I_{sc}\right)$$

$$\mathrm{C}_{\mathrm{V2}} = 0$$

Solving *R_s*, *n* and *I_{PH}* *with the assumption that* :

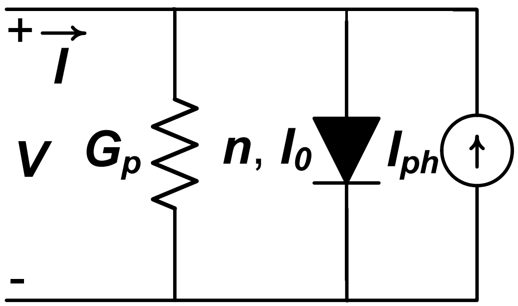
$$I_0 << I_{ph}$$

$$R_S = 2\,\mathrm{C}_{\mathrm{I2}}$$

$$I_{ph} = -\left(\,\mathrm{C}_{\mathrm{V1}} + \,I_{sc}\right)$$

$$n = \frac{\mathrm{C}_{\mathrm{I1}} - R_S\left(I_{ph} + I_{sc}\right)}{v_{th}}$$

7.3 Integration of the Co-content Function – *Particular cases*



$$C_{I2} = 0$$

$$C_{V2} = \frac{G_P}{2}$$

$$C_{I1} = +nv_{th}$$

$$C_{V1} = -\left(I_0 + I_{ph} + I_{sc}\right) - nv_{th}G_P$$

Solving R_s, n and I_{PH} ***with the assumption that :***

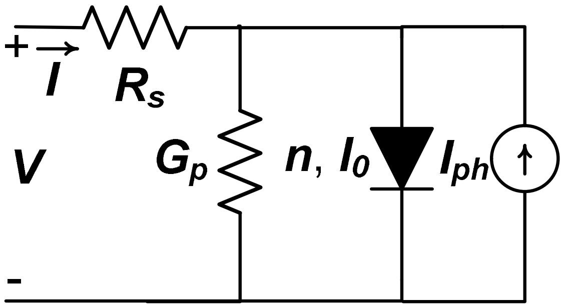
$$I_0 << I_{ph}$$

$$G_P = 2\ C_{V2}$$

$$n = \frac{C_{I1}}{v_{th}}$$

$$I_{PH} = -C_{V1} - nv_{th}G_P + I_{sc}$$

7.4 Integration of the Content Function [PE13]



Following the previous ideas [OR06], but integrating the Content instead of the Co-content:

$$\boxed{C(I,V) \equiv \int_0^I V\, dI}$$

Then the algebraic expression is obtained

$$C(I,V) = \frac{1}{2A}\left[\left(-V - B\,I + C\right)^2 - \left(-V + C\right)^2\right] - \frac{B\,I^2}{2} + A\,D\,I$$

where:

$$A = \frac{1}{G_p} \qquad B = R_S + \frac{1}{G_p} \qquad C = n\,v_{th} + \frac{I_{PH} + I_0}{G_p} \qquad D = I_{PH} + I_0$$

This method requires measurements with a uniform increment of current in order to evaluate the integration.

[PE13] L. Peng, Y. Sun, Z. Meng, Y. Wang, Y. Xu, A new method for determining the characteristics of solar cells, Journal of Power Sources, 227, pp. 131-136, 2013. http://dx.doi.org/10.1016/j.jpowsour.2012.07.061

7.5 Métodos usando pocos puntos: V_{OC} , I_{SC} , V_{max} , I_{max} .

Un excelente artículo de revisión fue presentado en 2019 [BA19].

Table 1. Main attributes of the 17 non-iterative parameter extraction methods.

No	Ref.	Method	Year	Number of Parameters	Eval Steps	Datasheet Sufficient?	Input Data
1	[14]	Phang	1984	5	7		$I_{sc}, V_{oc}, I_{mp}, V_{mp}, R_{sho}, R_{so}$
2	[15]	Sera	2008	4	4	✓	$I_{sc}, V_{oc}, I_{mp}, V_{mp}$
3	[16]	Saleem	2009	5	7		$I_{sc}, V_{oc}, V_{mp}, I_{60}, V_{60}$
4	[17]	Saloux	2011	3	3	✓	$I_{sc}, V_{oc}, I_{mp}, V_{mp}$
5	[12]	Accarino	2013	5	6	✓	$I_{sc}, V_{oc}, I_{mp}, V_{mp}, \alpha_{Isc}, \beta_{Voc}, V_{oc0}$
6	[18]	Khan	2013	5	7		$I_{sc}, V_{oc}, I_{mp}, V_{mp}, R_{sho}, R_{so}$
7	[19]	Cubas1	2014	5	8		$I_{sc}, V_{oc}, I_{mp}, V_{mp}, R_{sho}$
8	[19]	Cubas2	2014	5	8	✓	$I_{sc}, V_{oc}, I_{mp}, V_{mp}$
9	[56]	Cubas3	2014	5	9	✓	$I_{sc}, V_{oc}, I_{mp}, V_{mp}$
10	[20]	Bai	2014	5	11	✓	$I_{sc}, V_{oc}, I_{mp}, V_{mp}$
11	[21]	Aldwane	2014	4	4	✓	$I_{sc}, V_{oc}, I_{mp}, V_{mp}$
12	[22]	Cannizzaro	2014	4	8 or 11	✓	$I_{sc}, V_{oc}, I_{mp}, V_{mp}$
13	[23]	Toledo	2014	5	13		$I_{sc}, V_{oc}, I_{mp}, V_{mp}, I_{xx}, R_{sho}$
14	[24]	Louzazni	2015	5	7		$I_{sc}, V_{oc}, I_{mp}, V_{mp}, R_{sho}, R_{so}$
15	[25]	Batzelis	2016	5	8	✓	$I_{sc}, V_{oc}, I_{mp}, V_{mp}, \alpha_{Isc}, \beta_{Voc}, V_{oc0}$
16	[26]	Hejri	2016	5	5	✓	$I_{sc}, V_{oc}, I_{mp}, V_{mp}$
17	[27]	Senturk	2017	5	7	✓	$I_{sc}, V_{oc}, I_{mp}, V_{mp}$

[BA19] Batzelis, E., "Non-Iterative Methods for the Extraction of the Single-Diode Model Parameters of Photovoltaic Modules: A Review and Comparative Assessment," Energies, 12(3), pp. 358, 2019. DOI: 10.3390/en12030358

•Los métodos basados en pocos puntos de las características $I(V)$ ofrecen la ventaja de la rapidez y de poder ser aplicados directamente a las hojas de especificaciones técnicas de los fabricantes [1].

•Los métodos basados en muchos puntos ofrecen mayor precisión y requieren mayor cálculo numérico [2].

[1] E. Batzelis, Non-iterative methods for the extraction of the single-diode model parameters of photovoltaic modules: A review and comparative assessment, Energies, 2019, 12(3),358. . https://doi.org/10.3390/en12030358

[2] Ortiz-Conde, A. et al., 2014. A review of diode and solar cell equivalent circuit model lumped parameter extraction procedures. Facta Universitatis-Series: Electronics and Energetics, 27, 57-102. https://doi.org/10.2298/FUEE1401057O

Electrical Specifications

Module Type	OSA-144M-440		OSA-144M-445		OSA-144M-450	
Testing Condition	STC	NOCT	STC	NOCT	STC	NOCT
Rated output (Pmp/Wp)	440	330	445	334	450	338
Rated voltage (Vmp/V)	40.9	38.0	41.1	38.2	41.3	38.4
Rated current (Imp/A)	10.76	8.66	10.83	8.73	10.90	8.79
Open circuit voltage (Voc/V)	49.8	45.6	50.0	45.8	50.2	46.0
Short circuit current (Isc/A)	11.29	9.31	11.33	9.38	11.41	9.44
Module efficiency (%)	20.24%		20.47%		20.70%	
Power Tolerance (W)	0~+5					

Standard Test Condition (STC): Irradiance 1000W/m², Cell Temperature 25℃, AM1.5

Nominal Module Operating Temperature (NOCT): Irradiance 800W/m², Ambient Temperature 20℃, AM1.5, Wind Speed 1m/s

$$1)\ Open\ circuit \rightarrow V_{OC} \equiv V\big|_{I=0} \rightarrow (V_{OC},0) \quad 2)\ Short\ circuit \rightarrow I_{SC} \equiv I\big|_{V=0} \rightarrow (0,I_{SC})$$

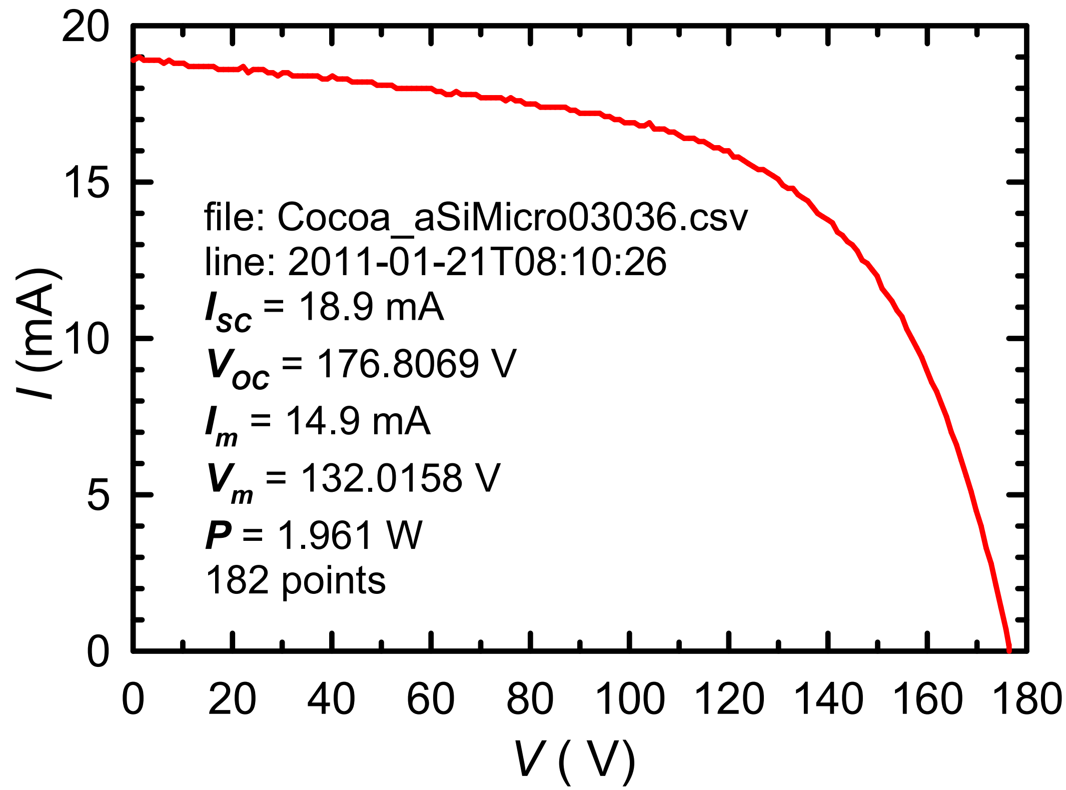
$$3)\ Rated\ voltage \rightarrow V_{\max} \quad and \quad Rated\ current \rightarrow I_{\max} \rightarrow (V_{\max},I_{\max})$$

4) (V_{max}, I_{max}) es el punto de máxima potencia.

Las 4 condiciones anteriores permiten obtener hasta 4 parámetros.

A public set of more than one million *I-V* characteristics, from NREL (National Renewable Energy Laboratories), is available at [MA14]

It contains *I-V* curves measured over one-year period in the USA for 22 PV modules.
It includes six different PV technologies: single- (c-Si) and multi-crystalline (mc-Si), CdTe, CIGS, HIT and amorphous silicon (a-Si) (crystalline, tandem, and triple-junction).
Each *I-V* characteristics contains more than 180 points.
The total dataset is larger than 1 GB.



[MA14] Marion, B., et al., "New data set for validating PV module performance models," IEEE 40th Photovoltaic Specialist Conference (PVSC), 2014.
DOI: 10.1109/pvsc.2014.6925171
For access to the data, please contact: Bill Marion at bill.marion@nrel.gov .

7.5.1 Método de Phang [PH84], de 5 parámetros, usando: V_{OC} , I_{SC} , V_{max} , I_{max} y las derivadas en I_{SC} y V_{OC}

Definiendo: $G_{po} \equiv \frac{1}{R_{sho}} \equiv \left.\frac{dI}{dV}\right|_{I=I_{SC}}$ $\qquad R_{so} \equiv \left.\frac{dV}{dI}\right|_{V=V_{OC}}$

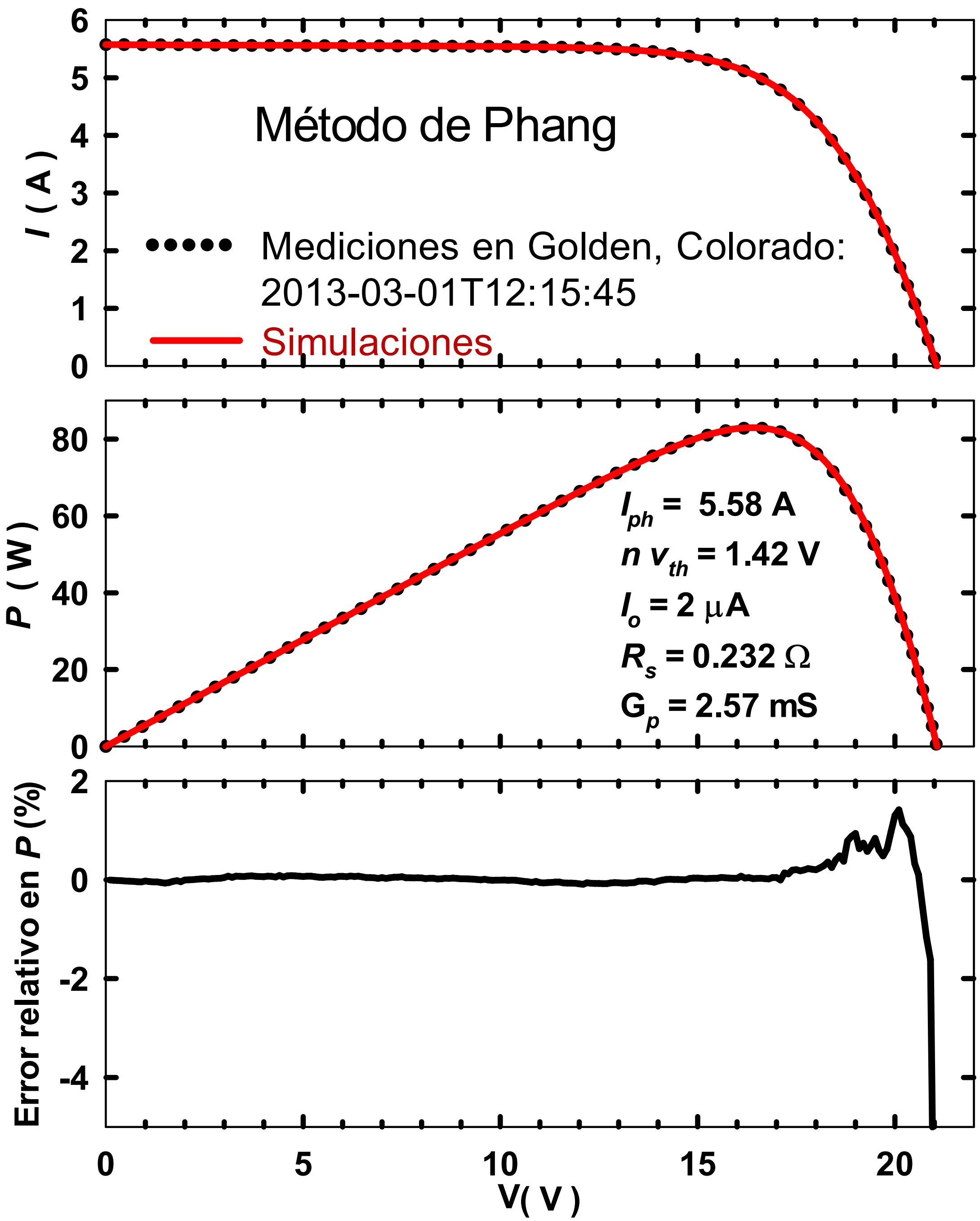
Aproximaciones: $R_{sh} \approx R_{sho}$ $\qquad R_s \approx R_{so} - \frac{n\,v_{th}}{I_o} \exp\left(\frac{V_{OC}}{n\,v_{th}}\right)$

$$n\,v_{th} \approx \frac{V_{\max} + R_{so} I_{\max} - V_{OC}}{\left[\ln\left(I_{SC} - V_{\max} G_p - I_{\max}\right) - \ln\left(I_{SC} - V_{OC} G_p\right) + \frac{I_{\max}}{I_{SC} - V_{OC} G_p}\right]}$$

$$I_{ph} \approx I_{SC}\left(1 + G_p R_s\right) + I_o\left[\exp\left(\frac{I_{SC} R_s}{n\,v_{th}}\right) - 1\right]$$

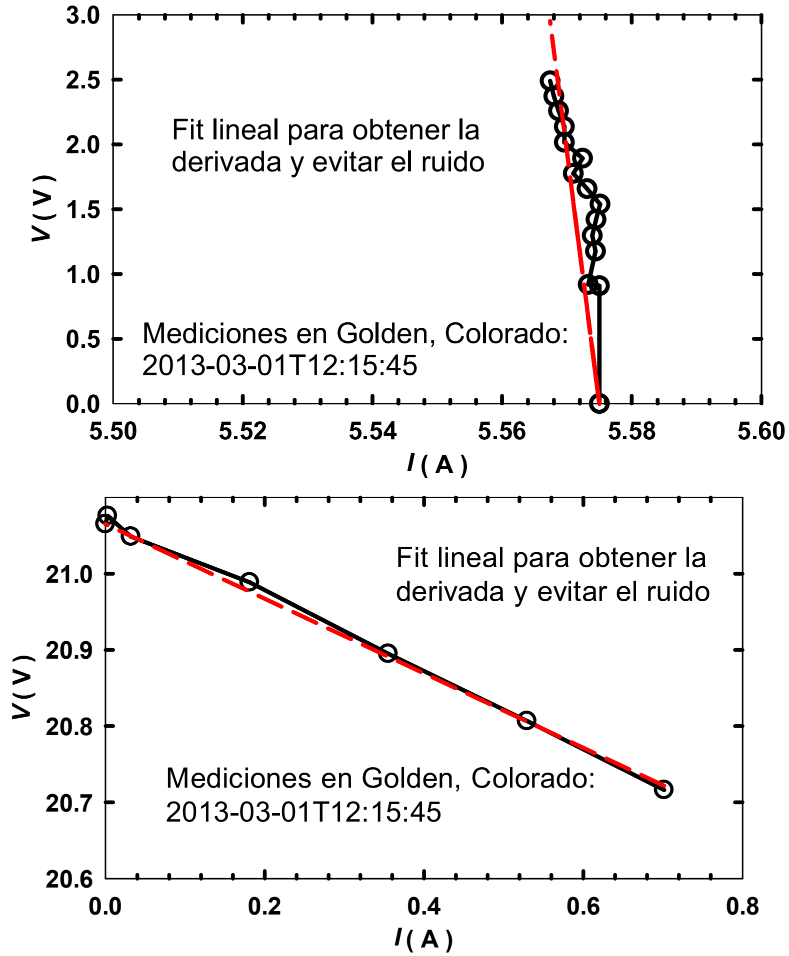
$$I_o \approx \left(I_{SC} - V_{OC} G_p\right) \exp\left(-\frac{V_{OC}}{n\,v_{th}}\right)$$

[P84] Phang, J.C.H., Chan, D.S.H., Phillips, J.R., "Accurate analytical method for the extraction of solar cell model parameters," Electron. Lett. 20 (10), 406-408, 1984. http://dx.doi.org/10.1049/el:19840281



El ruido en las mediciones dificulta la obtención de las derivadas. Debemos usar "fitting" de rectas para derivar.

$$G_{po} \equiv \frac{1}{R_{sho}} \equiv \left.\frac{dI}{dV}\right|_{I=I_{SC}} \qquad R_{so} \equiv \left.\frac{dV}{dI}\right|_{V=V_{OC}}$$



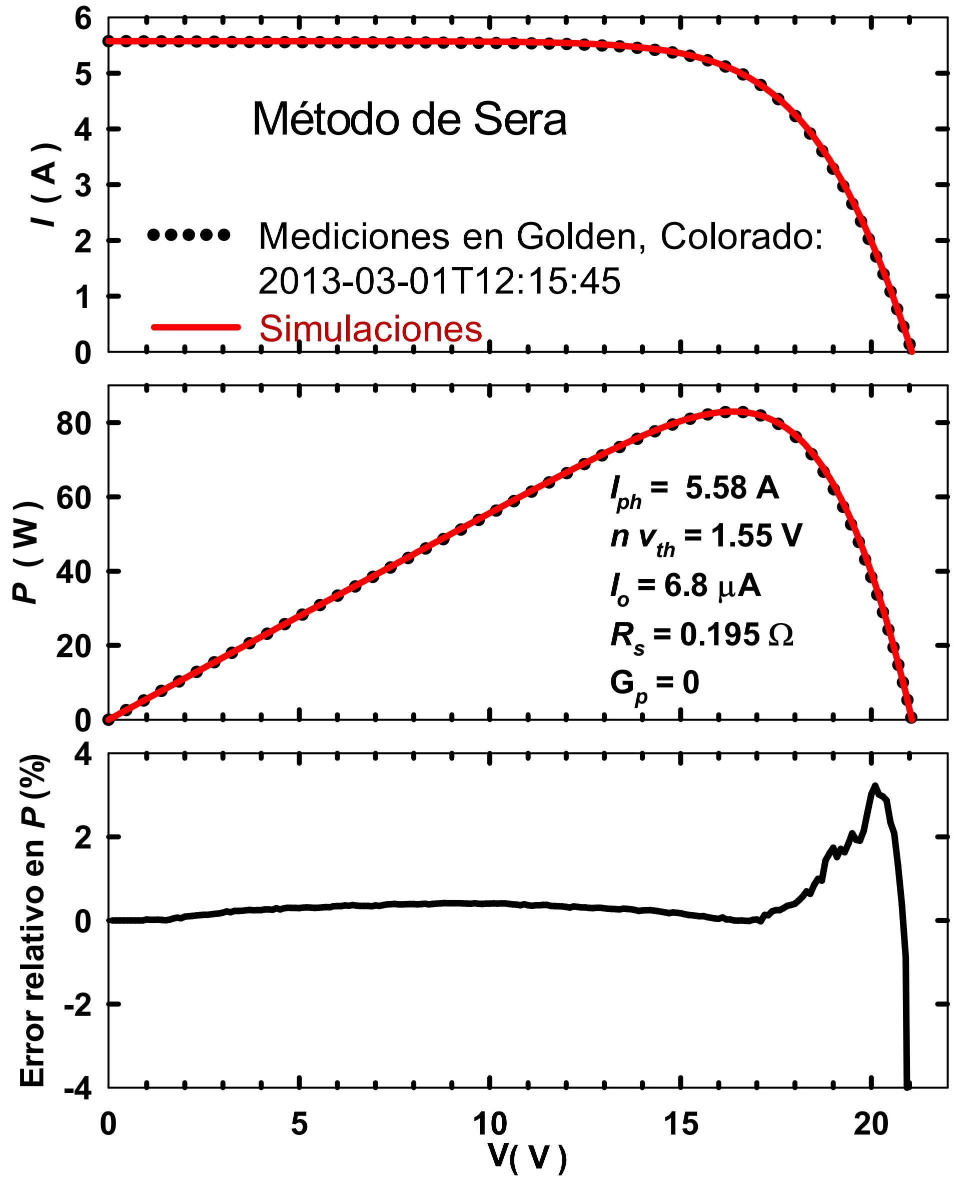
7.5.2 Método de Sera [SE08], con 4 parámetros, usando: V_{OC} , I_{SC} , V_{max} , I_{max}

$$I_{ph} \approx I_{SC}$$

$$n\,v_{th} \approx \frac{2V_{\max} - V_{OC}}{\left[\ln\left(\frac{I_{SC} - I_{\max}}{I_{SC}}\right) + \frac{I_{\max}}{I_{SC} - I_{\max}}\right]}$$

$$R_s \approx \frac{n\,v_{th} \ln\left(\frac{I_{SC} - I_{\max}}{I_{SC}}\right) + V_{OC} - V_{\max}}{I_{\max}}$$

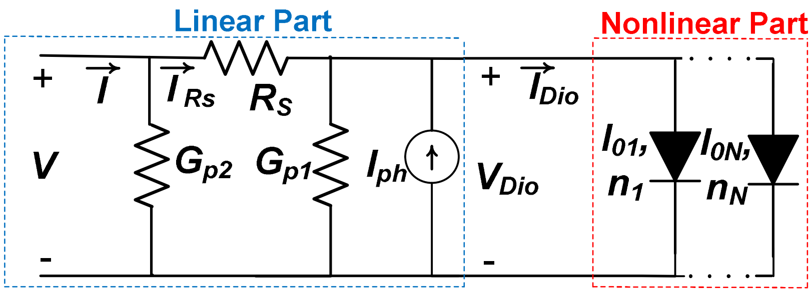
$$I_o \approx I_{SC}\ \exp\left(-\frac{V_{OC}}{n\,v_{th}}\right)$$



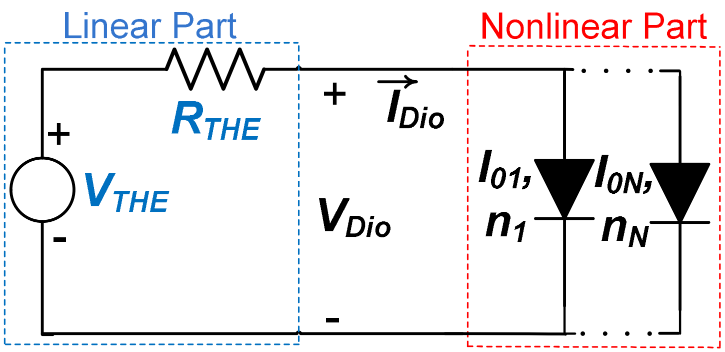
[SE08] Sera, D.; Teodorescu, R.; Rodriguez, P. Photovoltaic module diagnostics by series resistance monitoring and temperature and rated power estimation. In Proceedings of the 2008 34th Annual Conference of IEEE Industrial Electronics, Orlando, FL, USA, 10–13 November 2008; pp. 2195–2199. http://dx.doi.org/10.1109/IECON.2008.4758297

8. Multiple-exponential solar cell models

8.1 Alternative multi-exponential model for solar cells with resistances [OR12]

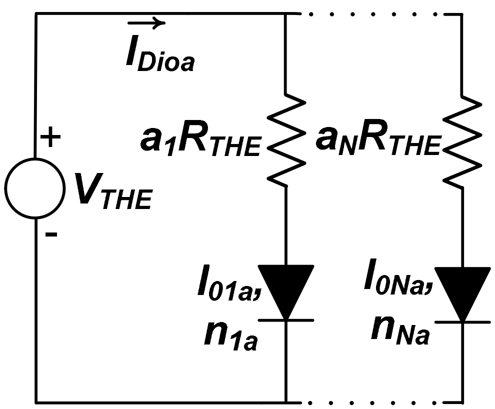


$$I = \left\{ \sum_{k=1}^{N} I_{0k} \left[\exp\left(\frac{V - R_s\left(I - G_{p2}V\right)}{n\, v_{th}} \right) - 1 \right] \right\} - I_{ph} + G_{p2}V\left(1 + G_{p1}R_s\right) + G_{p1}\left(V - R_s I\right)$$



Applying Thevenin's theorem to the linear components:

$$R_{THE} = \frac{R_s}{1 + G_{p1}R_s} \qquad V_{THE} = \left(\frac{V}{R_s} + I_{ph} \right) R_{THE}$$



By solving each branch separately and adding the solutions, the alternative model presents analytical solution for the current:

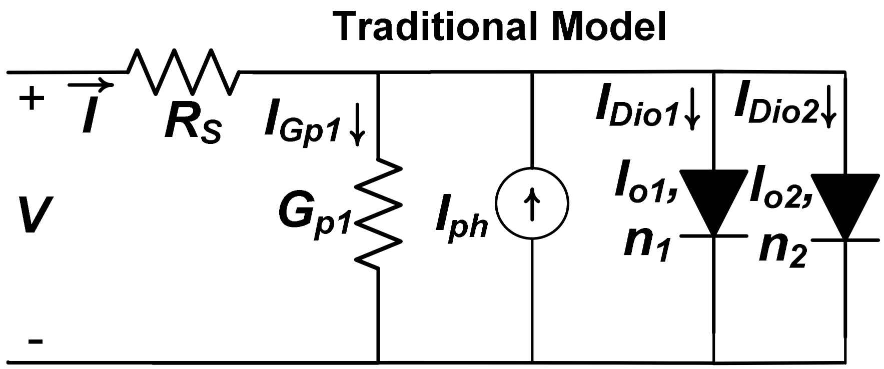
$$I_{Dioa} = \sum_{k=1}^{N} \left\{ \frac{n_{ka} v_{th}}{a_k R_{THE}} W_0 \left[\frac{a_k R_{THE} I_{0ka}}{n_{ka} V_{th}} \exp\left(\frac{V_{THE} + a_k R_{THE} I_{0ka}}{n_{ka} v_{th}} \right) \right] - I_{0ka} \right\}$$

where W_0 is the principal branch of the Lambert function.

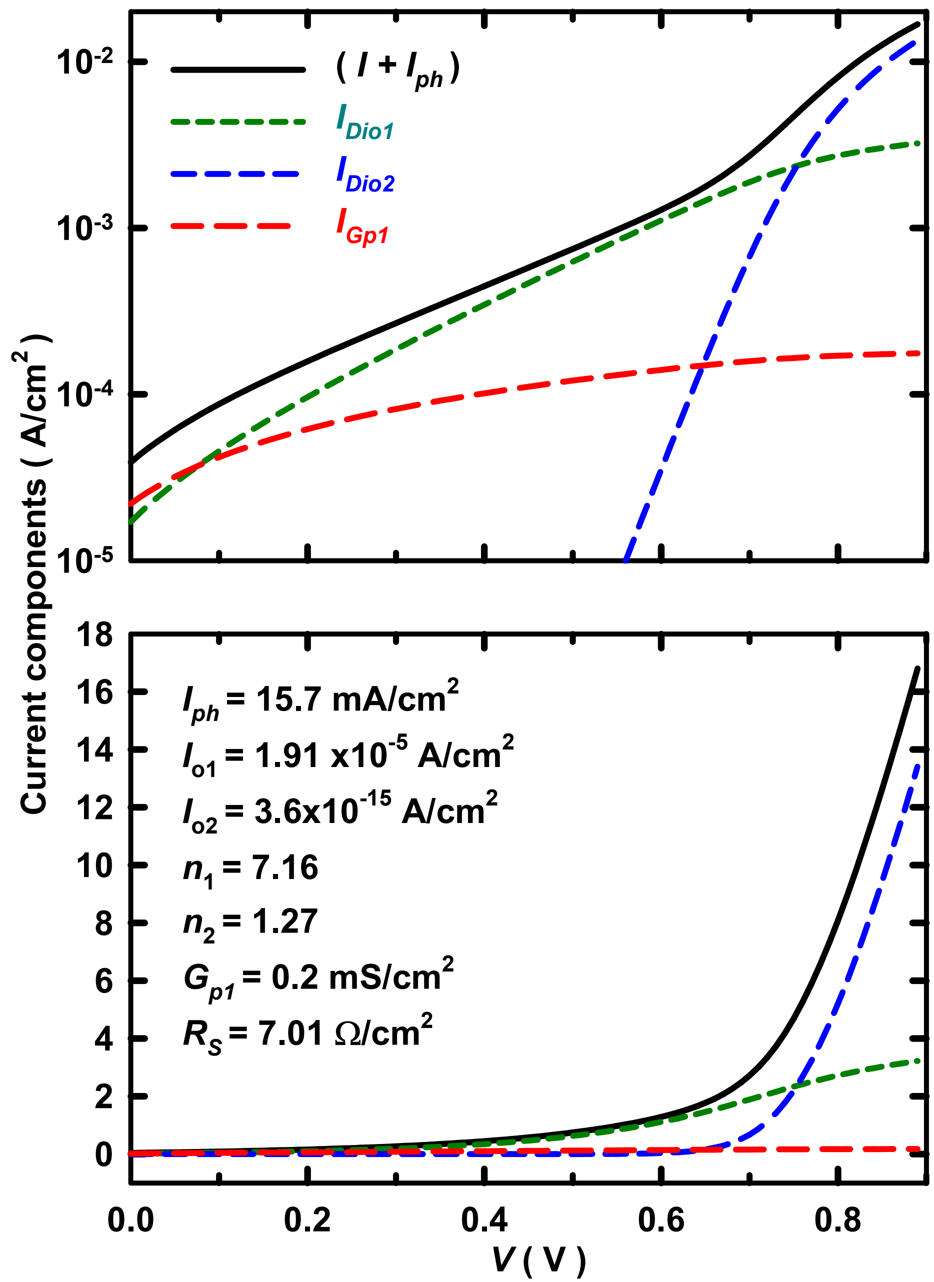
[OR12] A. Ortiz-Conde, D. Lugo-Muñoz and F. J. García Sánchez, " An explicit multi-exponential model as an alternative to traditional solar cell models with series and shunt resistances", IEEE Journal of Photovoltaics, Vol. 2, pp. 261-268, July 2012.
http://dx.doi.org/10.1109/JPHOTOV.2012.2190265

8.1 Alternative multi-exponential model for solar cells with resistances

Equivalent circuit of a ZnO/CdS/$CuGaSe_2$ solar cell studied by Saad and Kassis [SA11]



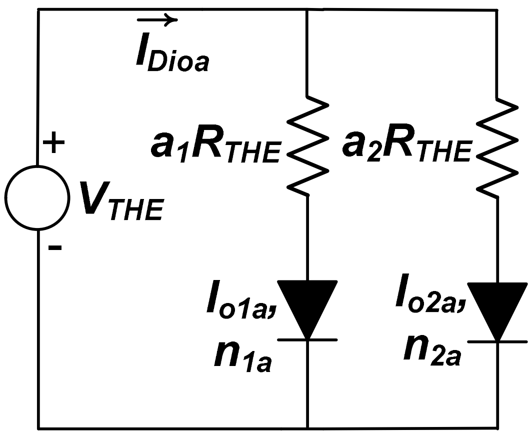
$$I = I_{01}\left[\exp\left(\frac{V - R_S I}{n_1 v_{th}}\right) - 1\right] + I_{02}\left[\exp\left(\frac{V - R_S I}{n_2 v_{th}}\right) - 1\right]$$
$$- I_{ph} + G_{P1}\left(V - R_S I\right)$$



[SA11] M. Saad and A. Kassis, “Current–voltage analysis of the record-efficiency CuGaSe2 solar cell: Application of the current separation method and the interface recombination model,” Sol. Energy Mat. Sol. Cells, vol. 95, pp. 1927–1931, 2011. http://dx.doi.org/10.1016/j.solmat.2011.02.022

8.1 Alternative multi-exponential model for solar cells with resistances

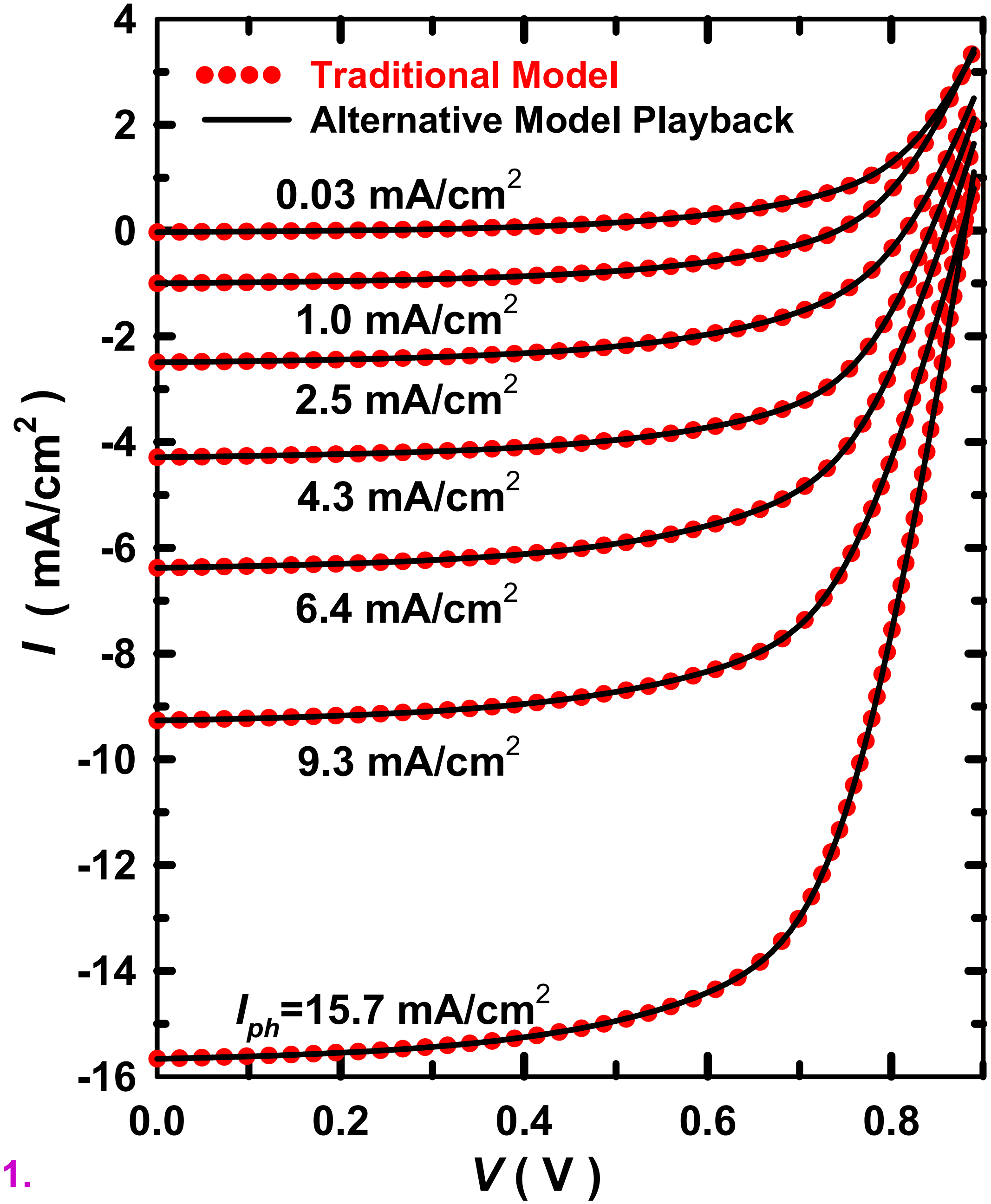
Alternative model's equivalent circuit



$$I_{Dioa} = \frac{n_{1a} v_{th}}{a_1 R_{THE}} W_0 \left[\frac{a_1 R_{THE} I_{01a}}{n_{1a} V_{th}} \exp\left(\frac{V_{THE} + a_1 R_{THE} I_{01a}}{n_{1a} v_{th}} \right) \right] - I_{01a}$$

$$\ldots + \frac{n_{2a} v_{th}}{a_2 R_{THE}} W_0 \left[\frac{a_2 R_{THE} I_{02a}}{n_{2a} V_{th}} \exp\left(\frac{V_{THE} + a_2 R_{THE} I_{02a}}{n_{2a} v_{th}} \right) \right] - I_{02a}$$

$$I = G_{p2} V + G_{p1} V_{Dio} - I_{ph} + I_{Dio}$$



[SA11] M. Saad and A. Kassis, "Current–voltage analysis of the record-efficiency CuGaSe2 solar cell: Application of the current separation method and the interface recombination model," Sol. Energy Mat. Sol. Cells, vol. 95, pp. 1927–1931, 2011. http://dx.doi.org/10.1016/j.solmat.2011.02.022

Conclusiones:

- **Hemos estudiado críticamente muchos métodos de extracción de parámetros de uniones p-n y celdas solares.**
- **Los métodos basados en la integración numérica son muy eficientes en la reducción del ruido de las mediciones.**
- **Los métodos basados en pocos puntos son muy convenientes de usar pero son sensibles al ruido.**
- **Los métodos fueron probados con simulaciones o con mediciones.**

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Si desea una copia de esta presentación, por favor, escríbame a: ortizc@ieee.org